The Net Discount Rate: Logical Relations Among Present Value Variables

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Abstract

Six economic variables or subsets thereof appear as sub-opinions or assumptions in forensic economists' reports. While the net discount rate (NDR) is the key variable, it is typically not primitive; rather, it is typically derived from two or three of the other five variables, defined here as the nominal discount rate (R), the real discount rate (r), the nominal wage growth rate (G), the real wage growth rate (g), and the increase in prices, or inflation (pi). While some equations connecting these six variables are well known, this paper systematically studies all of the possible interdependencies, providing 12 binary equations in which a variable is determined by two other of the variables, and 12 tertiary equations, in which a variable requires that three of the six variables be specified. These equations permit the construction of a spreadsheet which allows the forensic economist to quickly check an opposing expert's opinions for internal consistency, to calculate the remaining variables from the stated assumptions or opinions (if possible), and, if impossible, to understand the under-specification. Implicit in this approach is the belief that, in an era with data on real (TIPS) and nominal interest rates as well as various historical and projected nominal and real wage growth rates, an economic expert should have consistent opinions about all six variables regardless of the primitives selected. Additionally, the very important distinction between using discrete time equations rather than continuous time equations is discussed.

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Introduction

This paper discusses comprehensively the logical relations among six important variables which find their way into forensic economists' reports. These variables are:

R, the nominal interest rate used in discounting future flows

G, the rate of overall wage growth in nominal terms of such future flows

r, the real, or inflation adjusted, interest rate for discounting future flows

g, the real, or inflation adjusted, wage growth rate for compounding future flows

pi, expected inflation, or its proxy, the inflation rate of a version of the consumer price index

NDR, the net discount rate

The Problem

Of these six variables, it is required to specify a set of three inputs from which the values of the other three outputs will be implied. However, not all sets of three inputs result in such a consistent set of economic variables. Some sets of three are internally redundant and thus insufficient to specify the remaining three variables; worse, these redundant sets will generally be internally inconsistent unless chosen to satisfy an equation. Enumerated in the rules set forth in this paper, permissible sets of variables are identified and the reasons that the remaining sets are impermissible are given. In the process, 12 binary and 12 tertiary functions have been highlighted which express the six variables above as functions of the smallest sets of input variables. A few of the binary functions are well known in economics and forensic economics and are discussed next. No published discussion of the tertiary functions could be found in the literature.

It turns out that the tertiary functions, interesting in their own right, are indispensable in creating a spreadsheet which 'magically' does the following tasks: A. It accepts any set of inputs and displays all of the consistent outputs

- either all of the remaining outputs or a subset thereof; B. If the set of inputs is inconsistent, it so indicates, and shows the nature of the inconsistency.

Well-known Binary Relations

Some of the relationships among these variables are well known, e.g.,

(1) NDR(R,G) = (R-G)/(1+G)

(2) NDR(r,g) = (r-g)/(1+g)

which follow directly from the definitional requirement of NDR. In more detail, since flows increase by the nominal factor 1+G and must be discounted by 1+R, we must have by definition 1/(1+NDR)=(1+G)/(1+R) which must equal (1+g)/(1+r) as well. Others think of these last relations as (1+NDR)*(1+g) = (1+r) and (1+NDR)*(1+G) = (1+R).

The Fisher equation (Gordon, 2000) links real and nominal interest rates and asserts that (1+r)*(1+pi)=1+R. This can be rewritten as:

(3) r(R,pi) = (R-pi)/(1+pi).

The reader interested in interpreting this equation within a macroeconomic model may consult Sargent (Sargent, 1972) to ponder whether this is a structural or reduced form equation - the intention here is instead to discuss the mathematical links among these variables. Finally, the theory of wage growth envisaged in these relations holds that nominal wage growth, G, may be decomposed into real wage growth, g, and expected inflation, pi, via 1+g = (1+G)/(1+pi). This equation may be written as

(4) g(G,pi) = (G-pi)/(1+pi).

The following reference and interpretation of the wage equation is offered. In the very instructive paper by Leslie, if his equation (8) at page 642 is rearranged, and his multi-good model simplified to the one good model of classic economic growth theory, using his definition of the real wage as wc = W/c where W is the money wage and c is the price of the consumption good, the (continuous time) equation, G = g+pi follows from his condition (1) (Leslie 1985). In fact, g equals his technological change divided by labor's share of output. The interpretation of these variables is clearly within a longer run context, and not meant to capture macroeconomic phenomena over a business cycle.

After reviewing an earlier draft of this paper, Dr. Jerry Miner of Syracuse University offered the comment that "As regards nominal wage increases, issues of expected inflation arise in a macroeconomic setting when discussion turns to the reasons for rigidity of nominal wages in response to reductions in aggregate demand. One explanation proposed for nominal wage rigidity is that nominal wage rates in long term labor contacts are based on expected inflation rates. In macroeconomic texts that deal with these matters there is usually one or another version of adaptive expectations that serve to provide a dynamic model for the formulation of inflationary expectations presented as an alternative to a rational expectations perspective. The bottom line is that conventional economic analysis as regards nominal wages contains no effective counterpart to the Fisher equation as regards nominal interest rates. Unlike interest rates, which have values for future dates (except for long-term labor contracts), wage rates are only contemporaneous; there are no wage rates for future dates." The theme of this paper is in agreement with the comment by Dr. Miner about the business cycle, so that (4) should be given a long run growth theory or rational expectations interpretation.

Of course, wage growth for an individual may be further modified by age earnings coefficients for each future year, if appropriate - the formulation above is at the macroeconomic or growth theory level of "a rising tide lifts all boats" and the tide consists of inflation and productivity.

No stand is taken in this article as to how the forensic economist decides which primitive set of variables is used. In practice, one sees many variants, and this exercise was motivated by the desire to quickly check for consistency of assumptions and to ascertain the implied values of the other variables in any conceivable circumstance. For example, one might see R and G computed from some historical period, with nothing more. While sufficient for NDR, one might be interested in r, g, and pi - one would then expect pi to be also computed from historical data, but one might alternatively specify r or g. For example, r might be specified from current market (TIPS) data, g might be available from a government or private forecast, in which case NDR is determined; pi might be separately available, or derivable from current R data.

What does seem clear is that NDR is not itself a primitive - it is not directly observed, and economists therefore do not form professional opinions about it without reference to other variables. Provided here, in (1) and (2), are two ways about which an opinion concerning NDR may be derived, and below (in (21) and (22)) we offer the remaining two (new, and admittedly less frequent) alternatives. Further, it seems that a competent forensic economist should be able to say something about all six of these variables, despite the common observation that NDR, the variable driving the present value calculation, does not require pi if (R,G) or (r,g) are given. NDR does involve both (R,G) or (r,g) or more (per the equations to follow), which reinforces the first sentence of this paragraph.

A little algebraic substitution in the equations (1) - (4) results in the following 12 binary formulae which provide an expression for the left-hand side variable as a function of only the two inputs displayed as arguments and appearing on the right-hand side. What these equations say is that, given only the two variables on the right-hand side, there is sufficient information to determine the variable on the left-hand side. While (5) and (6) are well known, (7) through (12) appear to be new (Martin 2004)(Brown 2002). We note the symmetry - each of the six system variables appears determined by precisely two distinct binary relations.

Binary Formulae

(5)	pi(R,r)	=	(R-r)/(1+r)
(6)	pi(G,g)	=	(G-g)/(1+g)
(1)	NDR(R,G)	=	(R-G)/(1+G)
(2)	NDR(r,g)	=	(r-g)/(1+g)
(7)	G(R,NDR)	=	(R-NDR)/(1+NDR)
(8)	G(g,pi)	=	g+pi+(g*pi)
(3)	r(R,pi)	=	(R-pi)/(1+pi)
(9)	r(g,NDR)	=	g+NDR+(NDR*g)
(10)	R(r,pi)	=	r +pi+(r*pi)
(11)	R(G,NDR)	=	G+NDR+(G*NDR)
(12)	g(r,NDR)	=	(r-NDR)/(1+NDR)
(4)	g(G,pi)	=	(G-pi)/(1+pi)

These binary equations have three different uses: (1) the positive use of determining the opinion one must have about the left-hand side once the right-hand side variables have been determined; (2) the caution to the forensic economist NOT to specify the left-hand side variable unless it satisfies the equation in question; and (3) the caution of indeterminancy - specifying the three variables in any equation together will only determine those three variables, and none of the other three variables in the system.

We see that, of the 15 ways of selecting two variables from the set of six, 12 are accounted for by inspection of the four 'combining' groups of cycles of three elements below:

Combining:	Binary Reference:	Arguments of the Binary Equations Determining the third element via the Binary Equation:
Groups of 3	Equations	Groups of 2
(R,r,pi)	(5),(3),(10)	(R,r) (r,pi) (R,pi)
(pi,G,g)	(6),(8),(4)	(pi,G) (pi,g) (G,g)
(NDR,R,G)	(1),(7),(11)	(NDR,R) (NDR,G) (R,G)
(NDR,r,g)	(2),(9),(12)	(NDR,r) (NDR,g) (r,g)

It is natural to think of any choice of 3 of the 6 variables as arguments to a function which might determine one of the remaining 3 unknowns. There are 20 such triples, and to each triple, 3 variables could be determined. The 20 triples break down into a group of 12, a group of 4, and a final group of 4, in which each group has an interesting common property. We start by inspecting the 12 groups of 2 depicted above. Each pair requires a variable outside of its combining group in order to determine the remaining three variables. It turns out, upon inspection of the Tertiary Relationships below, that any such variable will do. This will have thus demonstrated the 'Calculation Rules.' If one chooses two variables from any of the four groups above, the remaining variable must be chosen from outside the group. If one chooses any other set of three variables, one has specified enough to consistently determine the rest of the system. For example, starting with (R,r), choosing g permits G (by (18) to follow), while pi and NDR follow from only two of the elements of the argument triple (R,r,g). Repeating this reasoning with each of the 11 other binary pairs produces, one at a time, the additional (11) tertiary functions displayed as (13)-(24).

On the other hand, the following pairs are NOT sufficient to determine any other variable in the system: 'Non-combining Groups' (R,g), (G,r), (NDR,pi). Intuitively, it is noted that the foregoing is a mix of nominal and non-corresponding real variables, so one should not expect them to combine to determine anything else. Despite their curse of inability to combine, they have more "spanning power" than the Combining Groups, since it will follow that ANY other choice of a third variable suffices to determine the system - one need not worry about under determinacy or over determinacy for these groups. Starting with the first of these, 4 choices are available, corresponding to any of the remaining unknowns; there result the triples (R,g,r),(R,g,pi),(R,g,G), and (R,g,NDR). These arguments (ignoring order) give rise to the tertiary functions (18), (21), (15) and (24) respectively. Repeating the reasoning with (G,r) and (NDR,pi) produces a second motivation for the twelve tertiary functions (13)-(24).

Without further ado, and with a little scratch paper, upon substitution using the binary rules for variables in different groups, the following tertiary relationship results hold:

(13) $R(NDR,g,pi) = NDR+(1+NDR)*(g+pi+(g*pi))$

(14) $R(G,g,r) = ((G-g)+r*(1+G))/(1+g)$

(15) $r(R,G,g) = ((R-G)+g*(1+R))/(1+G)$

(16) $r(G,NDR,pi) = ((G-pi)+NDR*(1+G))/(1+pi)$

(17) $G(r,pi,NDR) = (r*(1+pi)+(pi-NDR))/(1+NDR)$

(18) $G(g,R,r) = (g*(1+R)+((R-r))/(1+r)$

(19) $g(G,r,R) = (G*(1+r)-(R-r))/(1+R)$

(20) $g(R,pi,NDR) = ((R-pi)-NDR*(1+pi))/((1+pi)*(1+NDR))$

(21) $NDR(R,pi,g) = (((R-pi)-g*(1+pi))/((1+pi)*(1+g)$

(22) $NDR(r,pi,G) = (r+pi+(r*pi)-G)/(1+G)$

(23) $pi(G,NDR,r) = (G+NDR+(G*NDR)-r)/(1+r)$

(24) $pi(R,g,NDR) = ((R-g)-NDR*(1+g))/((1+g)*(1+NDR))$

Again, magically (symmetrically) each of the 6 system variables appears on the left-hand side in two and only two of equations (13) through (24). For example, (13) follows from R = NDR + G + (G * NDR), the (NDR,R,G) group in (11) - and substituting out G from (8); notice that the equation used, (8), came from another group, (pi,G,g). The other tertiary equations follow similarly.

These 12 formulae capture the ways that three different variables outside of the 'combining groups' may combine non degenerately to determine a remaining variable. Further, if a variable is not among the three inputs, then it may be calculated from those three inputs, either in the equation in question or from binary equations. In this sense, the binary and tertiary equations are sufficient for efficient calculations in this system. For example, consider in (24) the triple (R,g,NDR): once pi is directly determined from the tertiary equation (24), the two remaining variables, r and G, follow from binary equations (7) and (9) respectively. The 12 binary equations contain no set of (unordered) arguments more than once, and represent the 12 ways (out of 20) that three objects may be selected from a group of six by starting with the three 'non-combining pairs' above and selecting the four remaining variables for each. In other words, beginning with (R,g), there are four possible triples; beginning with the second non-combining group, (G,r), there are four more such triples, and similarly for (NDR,pi).

Returning to the (R,r,pi) group above, if (R,r) are specified, pi cannot be specified, and G, g and NDR remain. Choosing, say, G one sees that only NDR and g remain; (22) determines NDR while (19) determines g. Had g been specified, G would have followed from (18) and NDR from (21). Finally, had NDR been specified, G would follow from (17) while g follows from (20). Similarly, beginning in any of the other three Ggroups of 3, the choice of one variable from outside the group yields the remaining variables.

Now consider the first of the Non-Combining Groups, (R,g). These arguments occur in equations (15), (18), (21), and (24) above, which determine, respectively, r, g, NDR and pi as soon as, respectively, G, r, pi, and NDR are given. So, pick G. (15) determines r, (18) now determines g, while pi follows from (6) and NDR from (1). Had one picked r, pi, or NDR, the use of the tertiary and binary equations would have permitted solution for the other variables.

Of the 20 possible unordered triples, we have seen that the 4 "combining" groups do not permit anything outside of the group itself to be determined, and that the triple contains a redundancy – any two arguments determine the third. The 12 unordered triples yield a tertiary equation, through either of two motivations. What about the 4 remaining ordered triples unaccounted for? They are (R,r,NDR), (R,pi,G), (pi,r,g) and (R,pi,G). These arguments may vary independently. However, unlike the 12 triples, each of which generated one new function, for these pairs, *all three* of the ways of selecting two of the three arguments result in binary equations, so they contribute no new equation. In other words, pi(R,r,NDR) does not depend on NDR, and so the potentially tertiary function degenerates to a binary function. The argument may be repeated 11 more times, completing the discussion of the structural aspects of the problem.

The Calculation Box

Evidently, it never takes more than three arguments to specify a variable, although it may take as few as two, and some sets of three arguments may be redundant or inconsistent. It would be useful to have a 'Calculation Box' in a spreadsheet, into which one could enter any three variables, and immediately see the implications - either the rest of the implied system, or the redundancy or inconsistency. The 'Calculation Box' in a spreadsheet permits this. It exploits all of the 24 equations displayed thus far in this paper. This workbook, in Excel, can be downloaded at the following internet address: http://nafe.net/ . An example of the 'Calculation Box' spreadsheet is inserted as Table 1 so that the reader may see how the variables relate. In this example, the values of R, g, and pi are entered as the input values. The results are found in the "Consistent Set" column where the values of all six variables are shown. Data are entered only in the column titled "Input Value." All other values are automatically derived within the calculation box. Note that in Table 1 the "Output Value" for (r) was found by using the formula in the column titled "Binary 1." Likewise, the value for (G) was found in "Binary 2" and the value for NDR was found in "Tertiary 2." The spreadsheet is designed so that the set of values entered in the "Input Value" column are used to locate the correct binary or tertiary formula that will derive the unknown output. The calculation of NDR was found when the spreadsheet identified the input variables as R, g, and pi. It then located the tertiary formula that would accept these three inputs as a consistent set and generate the NDR. In this example, the formula was found in the "Tertiary 2" column.

Table 1: Calculation Box

NET DISCOUNT RATE (N DR) CALCULATIONS

Variable Name	Input Value	Output Value	Consistent Set	Binary 1	Binary 2	Tertiary 1	Tertiary 2
R	5.1000%	0.0000%	5.1000%	0.0000%	0.0000%	0.0000%	0.0000%
r	0.0000%	2.9838%	2.9838%	2.9383%	0.0000%	0.0000%	0.0000%
G	0.0000%	3.7336%	3.7336%	0.0000%	3.7336%	0.0000%	0.0000%
g	1.6000%	0.0000%	1.6000%	0.0000%	0.0000%	0.0000%	0.0000%
NDR	0.0000%	1.3172%	1.3172%	0.0000%	0.0000%	0.0000%	1.3172%
pi	2.1000%	0.0000%	2.1000%	0.0000%	0.0000%	0.0000%	0.0000%

R = Nominal Discount Rate　　r = real Discount Rate
G = Nominal Growth Rate　　g = real Growth Rate
NDR = Net Discount Rate　　pi = CPI increase

BINARY FUNCTION RULES

S&M Rule Number	If your set contains:	Then the disallowed input is:	Reason (Binary Function)	Binary Function Number From Table 1 above
1	R and r	pi	pi=(R-r)/(1+r)	Binary 2
2	G and g	pi	pi=(G-g)/(1+g)	Binary 1
3	R and G	NDR	NDR=(R-G)/(1+G)	Binary 2
4	r and g	NDR	NDR=(r-g)/(1+g)	Binary 1
5	R and NDR	G	G=(R-NDR)/(1+NDR)	Binary 1
6	g and pi	G	G=g+pi+(g*pi)	Binary 2
7	R and pi	r	r=(R-pi)/(1+pi)	Binary 1
8	g and NDR	r	r=g+NDR+(NDR*g)	Binary 2
9	R and pi	R	R=r+pi+(r*pi)	Binary 2
10	G and NDR	R	R=G+NDR+(G*NDR)	Binary 1
11	r and NDR	g	g=(r-NDR)/(1+NDR)	Binary 2
12	G and pi	g	g=(G-pi)/(1+pi)	Binary 1

TERTIARY FUNCTION RULES

S&M Rule #	If your set contains:	Then the disallowed input is:	Reason (Tertiary Function)	Tertiary Function Number From Table 1 above
1	NDR, g, and pi	R	R=NDR+(1+NDR)*(g+pi+(pi*g))	Tertiary 1
2	G, g, and r	R	R=((G-g)+r*(1+G))/(1+g)	Tertiary 2
3	R, G, and g	r	r=((R-G)+g*(1+R))/(1+G)	Tertiary 1
4	G, pi, and NDR	r	r=((G-pi)+NDR*(1+G))/(1+pi)	Tertiary 2
5	r, pi, and NDR	G	G=(r*(1+pi)+(pi-NDR))/(1+NDR)	Tertiary 1
6	g, R, and r	G	G=(g*(1+R)+(R-r))/(1+r)	Tertiary 2
7	G, R, and r	g	g=(G*(1+r)-(R-r))/(1+R)	Tertiary 1
8	R, pi, and NDR	g	g=((R-pi)-NDR*(1+pi))/((1+pi)*(1+NDR))	Tertiary 2
9	R, pi, and g	NDR	NDR=((R-pi)-g*(1+pi))/((1+pi)*(1+g))	Tertiary 2
10	r, pi, and G	NDR	NDR=(r+pi+(r*pi)-G)/(1+G)	Tertiary 1
11	G, NDR, and r	pi	pi=(G+NDR+(G*NDR)-r)/(1+r)	Tertiary 1
12	R, g, and NDR	pi	pi=((R-g)-NDR*(1+g))/((1+g)*(1+NDR))	Tertiary 2

Negative Values

In theory pi, r, g, G, and NDR could go negative while R cannot. Various combinations of negative value inputs have been tested in the spreadsheet and it appears to yield the correct answer. Further, even with all positive number inputs, it is possible to derive a negative solution. For example, assume G is assigned an input value of 3.0% and pi is assigned a value of 4.0%. Using the binary formula (4) (G-pi)/(1+pi) we derive the value of g as -0.9615%. This illustrates that there may be periods when the purchasing power of wages fails to keep pace with increases in the consumer price index.

Discrete v. Continuous Time

It is assumed in this paper that time is discrete and the 24 equations so far derived are all based on discrete time. In continuous time, these equations simplify drastically. Indeed, many forensic economists who compute the NDR as R – G or as r - g may be unaware of the difference between the proper treatments in discrete versus continuous time and are making an approximation, or, possibly, are working with data at such a fine time interval that it does not matter. An important assumption is that one is advised to incorporate the discrete time approximations discussed here and in (Martin 2004)(Albrecht & Moorhouse 1989) – for one thing, it shows proper training. For the record, all 24 discrete time formulae do have simpler continuous time analogues as follows. Generally, terms involving 1/(1 + any argument) become 1, and (1 + anything) when multiplying a variable may be replaced by 1. For example, (1) becomes NDR(R,G) = R-G, which may be seen from writing the right-hand side of (1) as (R-G)*(1-G+G*G- G*G*G*. . .) and observing that all terms but the leading 1 represent 'higher order of the smalls' as the time period shrinks to 0.

While the equations do simplify drastically when continuous time is used, economists using NDRs in their work should have an understanding of why discrete time equations are generally employed. Continuous time takes on a value at every point in time, whereas discrete time is only defined at integer or fractional integer values of the time variable. While discrete time values can be easily stored and processed on a computer, it is impossible to store the values of continuous time for all points along the real continuum on the digital computers we now employ as there is an infinite number of such points. The standard one year interval may represent the only data we have, and may simply be a good enough approximation. The payoff in accuracy from working with weekly, biweekly or monthly pay checks would not produce much if any benefit, and would be both more costly to produce and likely to represent the kind of "elusive exactness" against which courts have rightly cautioned. To derive an accurate continuous time model without the detailed time series data with which to estimate such a model is ill-considered. Additionally, continuous time stochastic processes represent analytic and technical difficulties not yet introduced into forensic economics. Such continuous time models have had success in finance, where transactions data, although not literally continuous, occur so frequently that it is useful to model high frequency phenomena. The continuous time framework has appeal, but it rests on complicated stochastic specifications which have not been tested. If these assumptions are violated, the model may well be less useful than the straightforward discrete time models in current use. Simply put, the present value variables used in forensic economic evaluations should be based on discrete time and the use of the simplified continuous time formulation is not presently advised.

Continuous Time Results

The 24 equations that follow are the continuous time counterparts to the previously listed 24 discrete time equations.

(5)=	pi(R,r)	=	R-r
(6)=	pi(G,g)	=	G-g
(1)=	NDR(R,G)	=	R-G
(2)=	NDR(r,g)	=	r-g
(7)=	G(R,NDR)	=	R-NDR
(8)=	G(g,pi)	=	g+pi
(3)=	r(R,pi)	=	R-pi
(9)=	r(g,NDR)	=	g+NDR
(10)=	R(r,pi)	=	r+pi
(11)=	R(G,NDR)	=	G+NDR
(12)=	g(r,NDR)	=	r-NDR
(4)=	g(G,pi)	=	G-pi
(13)=	R(NDR,g,pi)	=	NDR+g+pi
(14)=	R(G,g,r)	=	G-g+r
(15)=	r(R,G,g)	=	R-G+g
(16)=	r(G,NDR,pi)	=	G-pi+NDR

(17)=	G(r,pi,NDR	=	r+pi-NDR
(18)=	G(g,R,r)	=	g+R-r
(19)=	g(G,r,R)	=	G-(R-r)
(20)=	g(R,pi,NDR)	=	(R-pi)-NDR
(21)=	NDR(r,pi,G)	=	(R-pi)-g
(22)=	NDR(r,pi,G)	=	r+pi-G
(23)=	pi(G,NDR,r)	=	G+NDR-r
(24)=	pi(R,g,NDR)	=	(R-g)-NDR

Conclusion

While it has been shown that the concerns of this paper reside in discrete time, all of the interesting relations among the six economic parameters appearing in forensic economists' personal injury and wrongful death models are displayed in both discrete and in continuous time. The 24 discrete time equations are basic in the sense that, in each equation, it is possible to obtain the left-hand side variable with the subset of the right-hand side variables displayed. Conversely, one could complicate any of the right-hand sides adding a third variable to a binary equation or a fourth variable to a tertiary equation, via substitution, but the result would be uninteresting because it could be simplified. These equations permit the analyst to succinctly investigate any proposed opinions for consistency and completeness.

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