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# Allocation of Worklife Expectancy and the Analysis of Front and Uniform Loading with Nomograms

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## I. Introduction

Years of worklife expectancy (WLE) computed for the commonly used Markov model need not be consecutive and immediate. Theoretically correct decomposition or allocation, as of the injury date, for which both forward and backward algorithms were given, appeared in Skoog (2001) and (2002). Unfortunately the underlying transition probabilities are required, and these are not generally published. The only current exception is Krueger (2004) who supplies inter-year labor force status tables which, when combined with mortality probabilities, enable a user to calculate transition probabilities.[1] The required calculations are more amenable to a subroutine or macro, and essentially require the user to re-create the worklife expectancy calculation. In practice, forensic economists often either front-load (assume that the WLE comes immediately) or uniformly load (spread the WLE over a larger number of years, such as to age 65, 66 or 67). It is then natural to ask about the biases or corrections to be associated with these two loading methods of allocation. This paper reviews the theory in Section II and extends it in Section IV, providing closed form mathematical expressions (as opposed to computational programming code) involving the primitives of the problem, one step transition probabilities. Most readers will prefer Section III, the extensive middle part of the paper, however. For each of the two commonly employed allocation methods, charts, or nomograms,[2] are offered which allow forensic economists to visually make a correction to the nearest half percent, or argue that an offsetting correction has been introduced already in a calculation. In practice the necessary corrections vary with active versus inactive status and the age, sex, and education of an individual. Use of these corrections obviates performing more complex and data-intensive analysis of the exact decomposition, where there is no age-earning profile; if present, such a profile would require further study or the more exact methods.

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[1]Krueger only provides transition probabilities for all males; but transition probabilities for males and females by education can be computed from his inter-year labor force status tables when combined with mortality probabilities.

[2]We use the term nomogram to emphasize the use of charts proffered in the paper as graphical calculating devices which approximate exact computations with the belief that often approximate answers will be appropriate and useful to forensic economists in the context in which they are offered. A more exact, but also more time consuming, calculation may be warranted in a particular situation after making an easy and quick bias determination from a nomogram.

## II. Notation for the Theoretical Problem

The worklife expectancy for a person, as reported in Skoog-Ciecka (2001a and 2001b) depends on initial status, age, education and sex. Separate tables are calculated for each initial status, education and sex, with a row in each table giving characteristics of the distribution, including its mean (worklife expectancy). We let *sex*=(*m,f*), *ed*=(*all educations, less than high school,…, graduate degree*), *x*=*exact age,* and *s*=(*active, inactive*) the initial labor force state. *Worklife expectancy* is given by ${}^{s}e_{x}^{a}(ed,sex)$. It will be useful on occasion to suppress the dependence on (*ed,sex*) or (*s,ed,sex*) where no confusion will arise. The decomposition (1) below shows the fraction ${}^{s}_{j}e_{x}^{a}$ of each future year, up to a terminal age *TA*, that will be spent in the labor force:

$$(1)\qquad {}^{s}e_{x}^{a} \equiv {}^{s}e_{x}^{a}(ed,sex) = \sum_{j=0}^{j=TA-1} {}^{s}_{j}e_{x}^{a}(ed,sex) \equiv \sum_{j=0}^{j=TA-1} {}^{s}_{j}e_{x}^{a}\ .$$

Equation (1) was originally derived in Skoog (2001 and 2002) as equation (4.4), which presented a derivation from standard forward equations; if one were computing many of these, backwards recursions, equation (7.2) of Skoog (2001 and 2002) should be employed for computational efficiency. Letting

$$E_{x} = \begin{pmatrix} {}^{a}e_{x}^{a} & {}^{i}e_{x}^{a} \\ {}^{a}e_{x}^{i} & {}^{i}e_{x}^{i} \end{pmatrix}$$

denote the expected times in the right superscripted state, starting in the left superscripted state at age *x*, the first row gives the worklife expectancies, starting active (the (1,1) element) and inactive (the (1,2) element). Let

$$\mathrm{P}_{x} = \begin{pmatrix} {}^{a}p_{x}^{a} & {}^{i}p_{x}^{a} \\ {}^{a}p_{x}^{i} & {}^{i}p_{x}^{i} \end{pmatrix}$$

denote the corresponding age *x* transition matrix. Then the multi-period transition matrix

$$(2)\qquad {}_{j}\Pi_{x} \equiv P_{x+j-1}P_{x+j-2}\cdots P_{x} \equiv \begin{pmatrix} {}^{a}_{j}\Pi_{x}^{a} & {}^{i}_{j}\Pi_{x}^{a} \\ {}^{a}_{j}\Pi_{x}^{i} & {}^{i}_{j}\Pi_{x}^{i} \end{pmatrix}$$

provides the probability of occupying the right superscripted state at exact age *x*+*j* starting in the left superscripted state at age *x*. On the midpoint transition assumption, it follows that the parts of the year over the age interval (*x*+*j*,*x*+*j*+1) spent in the right superscripted state, starting in the left superscripted state at age *x* referred to on the right hand side of (1) are given by

$$(3)\qquad {}_jE_x \equiv \begin{pmatrix} {}^a_je^a_x & {}^i_je^a_x \\ {}^a_je^i_x & {}^i_je^i_x \end{pmatrix} = \frac{1}{2}\,{}_j\Pi_x + \frac{1}{2}\,{}_{j+1}\Pi_x \equiv \frac{1}{2}P_{x+j-1}P_{x+j-2}\cdots P_x + \frac{1}{2}P_{x+j}P_{x+j-1}\cdots P_x\,,$$

where we take ${}_0\Pi_x = I$ and ${}_1\Pi_x = P_x$ .

In terms of the years of additional activity random variable $YA_x$ studied in Skoog-Ciecka (2001a, 2001b, and 2002), we may decompose this random variable as

$$(4)\qquad YA_x = {}_0YA_x + {}_1YA_x + \cdots + {}_{TA-x-1}YA_x\,,$$

where ${}_iYA_x$ indicates years of activity during age ($x+i,x+i+1$) and $TA$ is terminal age, say 111. This notation suppresses the initial state for brevity, because it is not required for present purposes; however, (6) below reflects this consideration. The first row of the ${}_jE_x$ matrix in (3) provides the expectations of the elements of the ${}_iYA_x$ random variables, starting active and inactive. Knowing the sum $YA_x$ is not enough to determine the value of the present value *random variable* $PV(x,NDR)$ which represents the present value of $1 for each future year of labor market activity if active,

$$(5)\qquad PV(x,NDR) = \sum_{j=0}^{j=TA-x-1} \frac{{}_jYA_x}{(1+NDR)^{j+.5}}\,,$$

since equation (5) shows that the timing of the years of activity matter, where *NDR* is the net discount rate.

Forensic economists often use the term "present value" when they mean *expected present value*; our notation at (5) and (6) makes this distinction clear. Here the present value random variable is constructed from discounted ${}_iYA_x$ random variables, the latter taking on discrete values of 0, ½ or 1 over each future age interval, depending on the sample path. If active at the beginning and end of a year, 1 is realized; if a switch between active and inactive occurs at mid-year, a .5 is realized, while no activity in the interval results in a 0.

The theoretically exact or correct expected present value calculation, involving the decomposition (1) follows from taking expectations in (5) above, and is:

$$(6)\qquad EPV_C = EPV_C(x,NDR,WLE) = EPV_C(x,NDR,s,ed,sex) = \sum_{j=0}^{j=TA-x-1} \frac{{}^s_je^a_x}{(1+NDR)^{j+.5}}.$$

The front-loading version or approximation of expected present value, assuming as we do throughout that we have mid-year transitions within the Markov model, is

$$EPV_{FL} = EPV_{FL}(x, NDR, WLE) = EPV_{FL}(x, NDR, s, ed, sex) =$$

(7) $$\sum_{j=0}^{j=[{}^s e_x^a(ed,sex)]-1} \frac{1}{(1+NDR)^{j+.5}} + \frac{{}^s e_x^a(ed,sex) - [{}^s e_x^a(ed,sex)]}{(1+NDR)^{[{}^s e_x^a(ed,sex)]+(\{{}^s e_x^a(ed,sex)-[{}^s e_x^a(ed,sex)]\}/2)}}$$

In (6) and (7), the C and FL refer to, respectively, "correct" and "front-loading." The last term in (7) requires special treatment. Since generally ${}^s e_x^a$ is not an integer, the numerator in the last term in (7) is taken to be ${}^s e_x^a$ -[${}^s e_x^a$], where [${}^s e_x^a$] is the greatest integer in ${}^s e_x^a$, i.e., it is the remaining part of the worklife expectancy attributable to the final year, rather than the full years reflected by the 1's in the terms in the summation sign. Similarly, since the earnings flow in the final term is over a fractional year, its midpoint is not on a half integer, as all previous terms are; it begins at $[{}^s e_x^a(ed, sex)]$ and ends at ${}^s e_x^a(ed, sex)$, accounting for the denominator in the last term in (7).

We may view the terms in (7) as discount factors times weights $w_j^2$ which vanish when, and after, $j$=[*WLE*+1] and are 1 for $j<WLE$. Similarly we may view (6) as consisting of weights $w_j^3$, say, which are ${}_j^s e_x^a$ , generally less than 1. Now

$$\sum w_j^2 = \sum w_j^3 = WLE \text{ and } w_j^2 > w_j^3 \text{ for } j<WLE,$$

where the $\frac{1}{(1+NDR)^{j+.5}}$ terms are larger than the $\frac{{}_j^s e_x^a}{(1+NDR)^{j+.5}}$ terms.

Also,

$$w_j^2 = 0 < w_j^3 \text{ for } j>WLE,$$

where the $\frac{0}{(1+NDR)^{j+.5}}$ terms are smaller than the $\frac{{}_j^s e_x^a}{(1+NDR)^{j+.5}}$ terms.

It follows that $EPV_{FL} > EPV_C$ provided $NDR > 0$. (We have omitted the discussion of the fractional year term at the end of (7) for brevity.)

That there is bias due to front-loading is generally appreciated, although without a handle on the theoretically correct weights ${}_j^s e_x^a$, no proof could be too rigorous. The same idea is also present in the well known actuarial inequality asserting that an annuity to life expectancy is worth more than an annuity for life: $a_{\overline{e_x}|} > a_x$ (see King, 1887, and Steffensen, 1919, for example).[3] The annuity to life expectancy employs weights of one up to life expectancy and zero thereafter, and in place of the ${}_j^s e_x^a$ weights are survival probabilities in the life annuity; *NDR* is replaced by the interest rate. Ben-Zion and Riddle (1985) studied the actuarial inequality primarily for deferred annuities in divorce, showing that as the deferral period grows, the bias changes sign and becomes

[3]This result also was recognized by de Witt in 1671 and by Nicholas Bernoulli in 1709 (Poitras, 2000) but perhaps only intuitively or empirically.

arbitrarily bad. Skoog and Ciecka (2007) provide a self contained proof of the above mentioned actuarial inequality, prove new equalities for medians and modes of annuities, inequalities linking means, medians and modes of the present value annuity probability mass functions, and graph deviations of $a_{\overline{e_x}|}$, $a_{\overline{median\ yearslived_x}|}$, and $a_{\overline{mode\ yearslived_x}|}$ from $a_x$, relating the inequalities to the skewness of the underlying survival probability mass function.

We define here the percentage correction for $EPV_{FL}$, the multiplicative factor by which the computed quantity must be adjusted to bring about the theoretically correct result, as

(8) $$pctcor_{FL}(x,NDR,s,ed,sex) = \frac{EPV_C(x,NDR,s,ed,sex) - EPV_{FL}(x,NDR,s,ed,sex)}{EPV_{FL}(x,NDR,s,ed,sex)} 100.$$

The uniform loading approximation spreads years of worklife expectancy equally to an age, typically beyond age plus worklife, such as age 65, 66 or 67, which we call *ULMAX*, the maximum age over which uniform loading spreads payments of $1. These choices correspond to the Social Security normal retirement age for persons born before 1938, between 1943 and 1954, and in 1960 and later, respectively. Our graphs employ age 66, both capturing most of the baby boomers and providing an age which will differ by at most 1 from the choice of most practitioners. Letting $WLE_x$ denote the worklife expectancy of either ${}^{a}e_x^a$ or ${}^{i}e_x^a$ as appropriate, the uniform allocation factor is defined as:

(9) $$uaf = \frac{WLE_x}{ULMAX - x}.$$

This factor or ratio is the first part of the expected present value of $1 per year for $WLE_x$ years, allocated not immediately as in front loading, but spread out as *uaf* per year, to age *ULMAX*. The expected present value under uniform loading is then

(10) $$\begin{aligned} EPV_{UL} &= EPV_{UL}(x,NDR,WLE,ULMAX) = EPV_{UL}(x,NDR,s,ed,sex,ULMAX) \\ &= \frac{WLE_x}{ULMAX - x} \sum_{j=0}^{j=ULMAX-x-1} \frac{1}{(1+NDR)^{j+.5}} \end{aligned}$$

We correspondingly define the percentage correction of $EPV_{UL}$ which must be added to it to produce $EPV_C$ as

(11)
$$pctcor_{UL}(x,NDR,s,ed,sex,ULMAX) = \frac{EPV_C(x,NDR,s,ed,sex) - EPV_{UL}(x,NDR,s,ed,sex,ULMAX)}{EPV_{UL}(x,NDR,s,ed,sex,ULMAX)} 100.$$

For younger ages, and especially beginning active and when larger *ULMAX* values are selected, uniform weighting typically understates the expected present value, so the latter correction factor is positive, while for all ages, front loading overstates the expected present value, resulting in negative percentage correction factors (assuming a positive *NDR*). We display both (8) and (11) for four *NDR* values on the same nomogram. All of our figures employ $ULMAX = 66$ .

For younger ages, *x*, $WLE_x < ULMAX - x$ , so that $uaf < 1$ . However, for *ULMAX* choices in the range of 65, 66 and 67, and for large enough *x* values, the inequality reverses. Since it makes no sense to attempt to allocate more than one year to a calendar year, as would be required when $uaf > 1$ , we truncate our graphs of $pctcor_{UL}(x, NDR, s, ed, sex, ULMAX)$ when $uaf = 1$ . In fact, we define $x_{FL}$ to be the solution to $WLE_x = ULMAX - x$ , or equivalently to $uaf(x) = 1$ . Therefore, when $uaf(x_{FL}) = 1$ , uniform loading is front loading; so this point, where the uniform loading graph ends, is on both uniform and front loading graphs.

While uniform loading often requires a positive adjustment, inspection of the nomograms shows that uniform loading in other cases can be too generous and thereby require a negative correction, especially for those initially inactive. Also, it is biased and too high for all ages for the highest education level for active males. The point where the sign of the correction changes varies with the choice of *ULMAX,* and can be made to occur beyond 65 for all of the active male groups provided we increase *ULMAX* to 75.

## III. Results

We present 24 pages of charts or nomograms: Figures 1-6 for active men, Figures 7-12 for inactive men, Figures 13-18 for active women, and Figures 19-24 for inactive women.[4] Each set of six nomograms begins by pooling all educational levels, and then proceeds through educational levels, beginning with "less than high school" and concluding with "graduate degree," with three intermediate educational levels between these extremes. The charts are nonparametric and simply graph percent corrections against age. Curves associated with uniform loading often appear with positive ordinates (percent corrections) at young ages, but curves depicting front loading always appear in the negative region. The highest and lowest curves, corresponding to extreme *NDR*'s (i.e., 1% and/or 4%) are indicated with additional captioning. Any sex, activity status, and education level triple identifies one of the 24 nomograms; the age at injury permits one to vertically read corrections or biases corresponding to four non-zero *NDR*'s for front loading and four *NDR*'s for uniform loading. Interpolation will be necessary for non-integral *NDR*'s. The results may easily be read to the nearest half percent or better.

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[4]Please go to http://fac.comtech.depaul.edu/jciecka/frontloadinguniformloadingnomograms.doc or http://www.legaleconometrics.com for online viewing in color. Viewing in color may enhance readability of the nomograms. Also, by double clicking, a nomogram will become live in the sense that a pointer will show the exact coordinates of any point on the chart. This eliminates the need to "eyeball" percent corrections.

Figure 25 and Figure 26 illustrate the detailed calculations that lie behind the percentage corrections in our nomograms. Figure 25 refers to front loading for active 22-year-old males with high school educations and worklife expectancy of 34.91 years (Skoog and Ciecka, 2001b). The "Correct Decomposition" function shows the correct allocation of worklife to individual years beyond age 22; these are the ${}_{j}^{a}e_{x}^{a}$ elements in ${}_{j}E_{x}$ in equation (3). Their corresponding present values are depicted as the points marked on the "PV of Correct Decomposition" function; their sum is \$23.98276 with $NDR = .02$. Front loading allocates one year of activity to each of the first 34 years from age 22 and the remaining .91 of worklife expectancy to the 35th year; this is the "FL" function. "PV of FL" shows the present value of the "FL" allocation; its sum is \$25.20423. The resulting percentage correction for front loading is [(\$23.98276 – \$25.20423)/\$25.20423]100 = - 4.85%. One sees the "PV of FL" always above the "PV of Correct Decomposition" for the first 34.91 years. The "PV of Correct Decomposition" shows increments to present value for about 60 years, but the overestimates in the "PV of FL" for the first 34.91 years more than offset the accumulation of present value increments in the correct decomposition after 34.91 years, and the net result of front loading is the overestimate 4.85% as shown in Figure 3. Uniform loading to age 66 allocates 34.91/(66 – 22) = .793 years of activity to each year from age 22, and nothing thereafter as shown by the "UL" line in Figure 26. "PV of UL" is the present value function for this allocation, a total of \$23.30438, again with $NDR = .02$. The percentage correction from uniform loading is [(\$23.98276 – \$23.30438)/\$23.30438]100 = 2.91%. Figure 3 shows approximately this amount must be added to present value if uniform loading were to be used.

The following list reiterates, summarizes and clarifies nomogram characteristics:

1. If the *NDR* were zero, neither front loading nor uniform loading requires any correction. Therefore, the proper correction curve for $NDR = 0$ coincides with the horizontal axis in all nomograms.

2. Front loading produces overestimates of correct expected present values; therefore, front loading curves are below the horizontal axis for all $x < TA - 1$ indicating negative adjustments to arrive at correct expected present values.

3. With front loading, the larger the *NDR*, the worse the overestimate of expected present value at any age. Overestimates approach zero as the *NDR* approaches zero.

4. Given age, gender, and education, front loading overestimates of expected present values typically are worse for inactives than actives (*e.g.*, compare Nomograms 1 and 7). In addition, inactive curves usually contain more local minima and maxima points (e.g., compare Nomograms 6 and 12 and also Nomograms 18 and 24).

5. Given age, initial status, and education, front loading corrections tend to be larger (in absolute value) for women than for men (e.g., compare Nomograms 1 and 13).

6. Uniform loading curves only are defined for ages $x$ such that $\dot{e}_x^a \leq ULMAX - x$ , otherwise more than one year of activity occurs between ages $x$ and $ULMAX$ when $\dot{e}_x^a > ULMAX - x$ .

7. Uniform and front loading correction curves intersect when $\dot{e}_x^a = ULMAX - x$ since, in this case, uniform loading allocates $\dot{e}_x^a / (ULMAX - x) = 1$ year of activity over each of $(ULMAX - x)$ years just as occurs with front loading. This property can be useful when reading complex nomograms where uniform and front loading curve intersect (see Nomogram 24).

8. Uniform loading can, and often does, produce underestimates (requiring positive corrections); but uniform loading can result in overestimates (requiring negative adjustments) as well. In fact, uniform loading adjustments can be negative throughout their entire domain (e.g., see Nomogram 6); and they can be negative initially, become positive, and then be negative again at later ages (e.g., see Nomogram 7).

9. When a uniform loading curve appears above the horizontal axis, the correct expected present value has been underestimated; and usually a worse underestimate results (read off a higher curve), and a larger positive correction percentage should be made, when discounting at a greater *NDR* (e.g., see Nomogram 1). However, this does not necessarily occur. Given age, labor force status, education, and gender, let $EPV_C(i)$ and $EPV_C(r)$ denote the correct expected present values at *NDR*'s $i$ and $r$, with $r > i$. Similarly, let $EPV_{UL}(i)$ and $EPV_{UL}(r)$ denote present values resulting from uniform loading with $i$ and $r$, respectively. In order for the correction under $r$ to exceed the correction based on $i$, that is $[(EPV_C(r) - EPV_{UL}(r)) / EPV_{UL}(r)] > [(EPV_C(i) - EPV_{UL}(i)) / EPV_{UL}(i)]$ ,
it follows that $EPV_C(r) / EPV_C(i)] > EPV_{UL}(r) / EPV_{UL}(i)$ .

   This condition usually is fulfilled but not necessarily so, for example see Nomogram 19 at age 17. Corrections are very small at this age and therefore differences between corrections are of no practical import. The nomogram curves indicate corrections in the neighborhood of .1% to .5%. In fact, the exact corrections are .119% when the *NDR* is .04 and .501% when the *NDR* is .02.

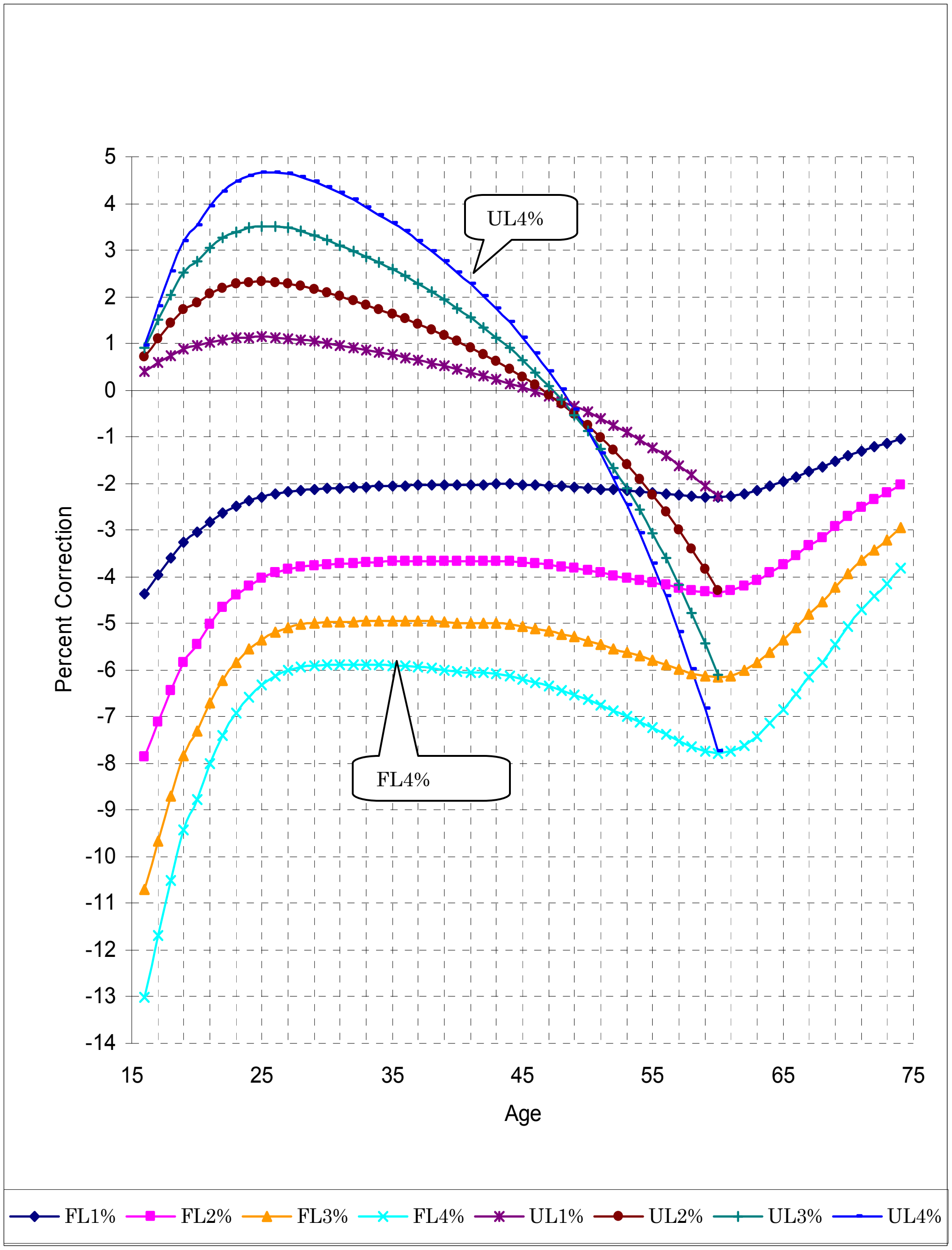


Figure 1. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Men, Regardless of Education

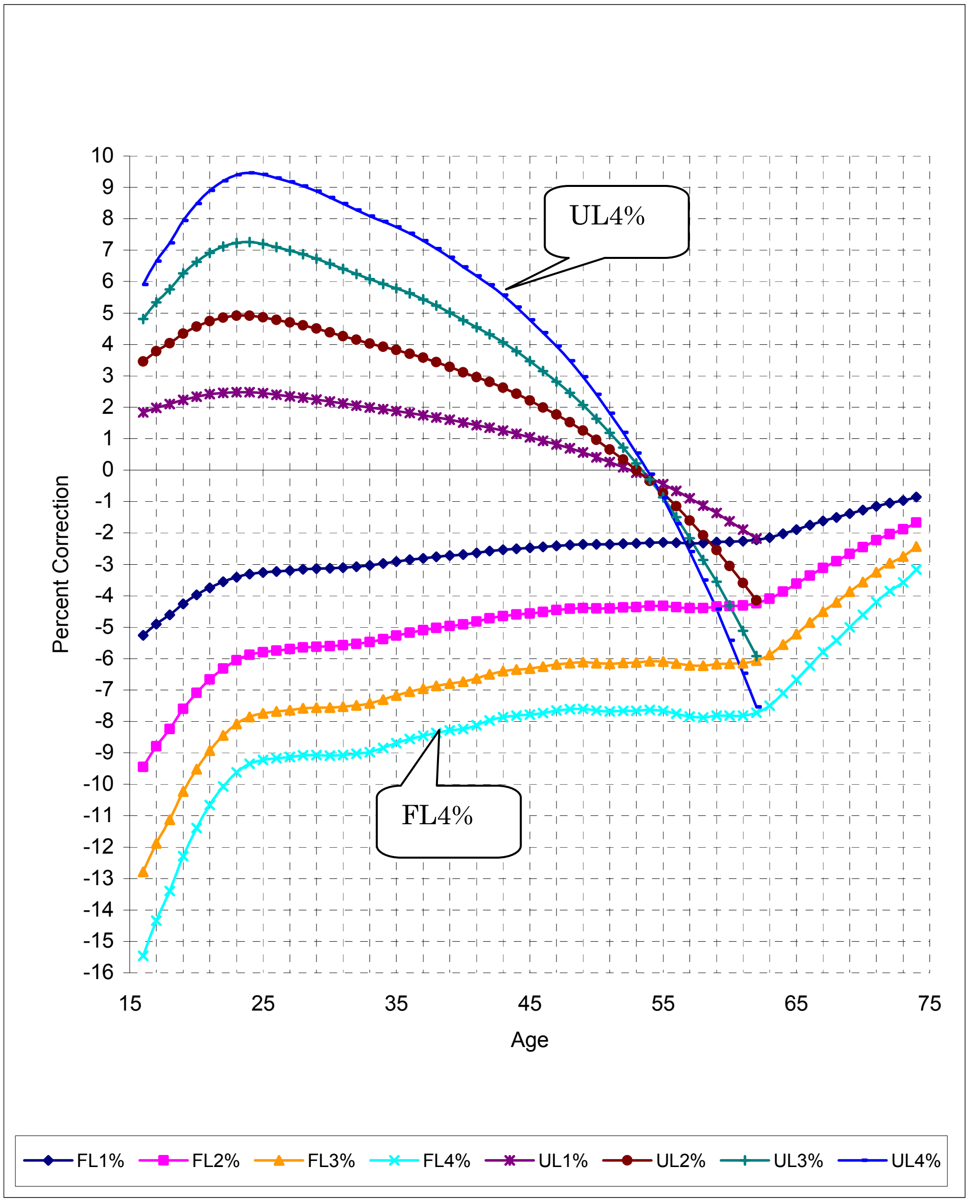


Figure 2. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Men with Less Than a High School Diploma

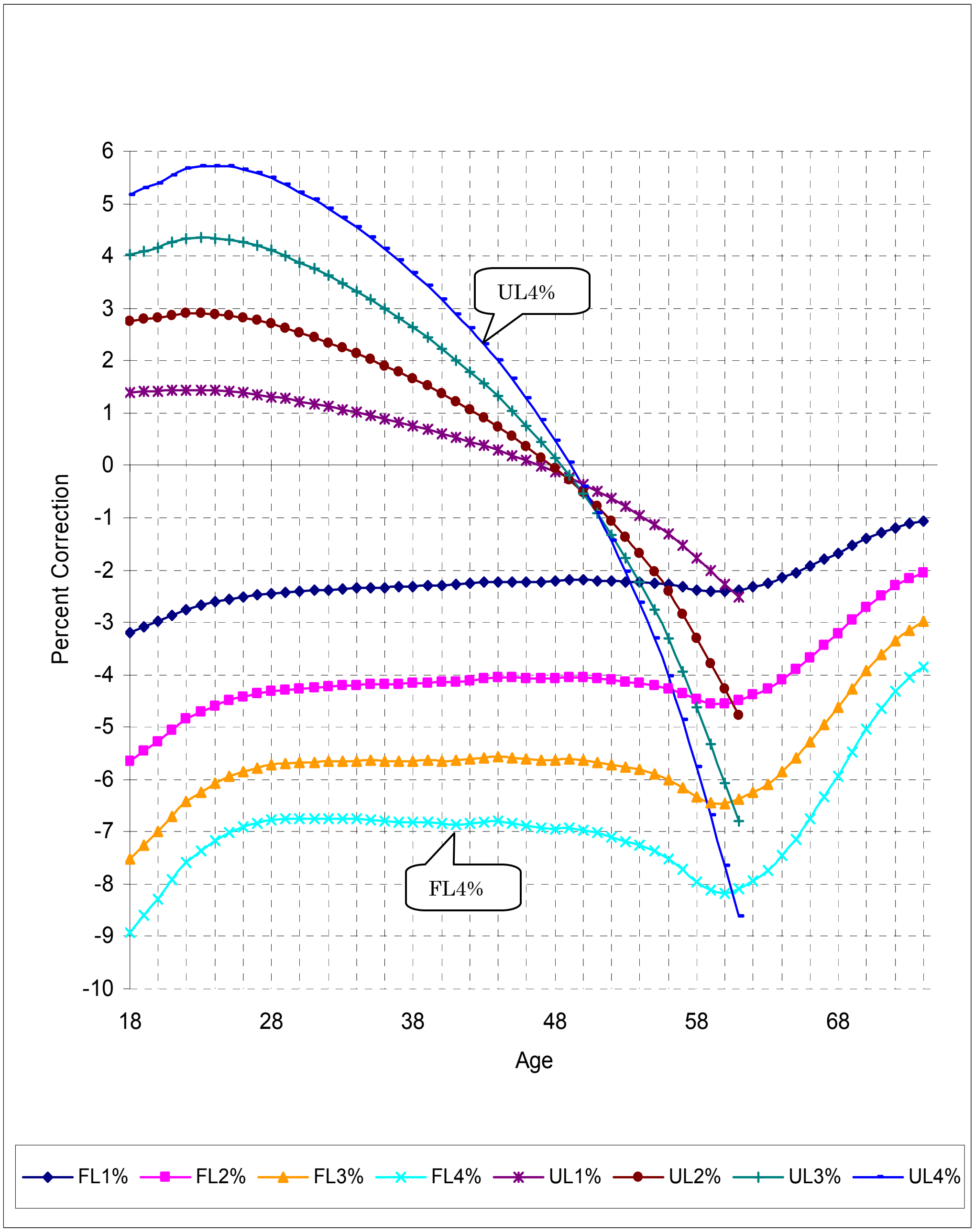


Figure 3. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Men with a High School Diploma Only

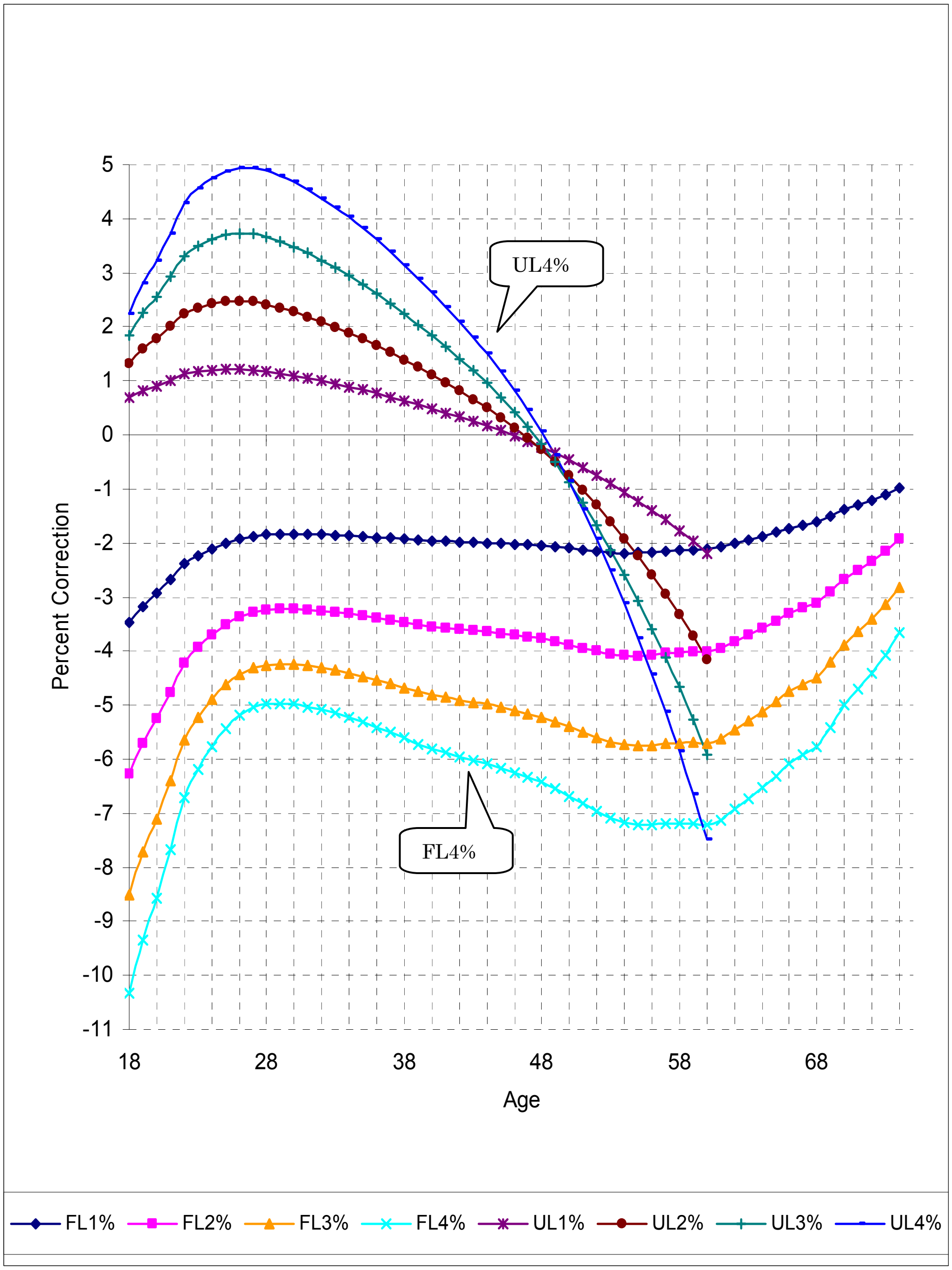


Figure 4. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Men with Some College But No Bachelor's Degree

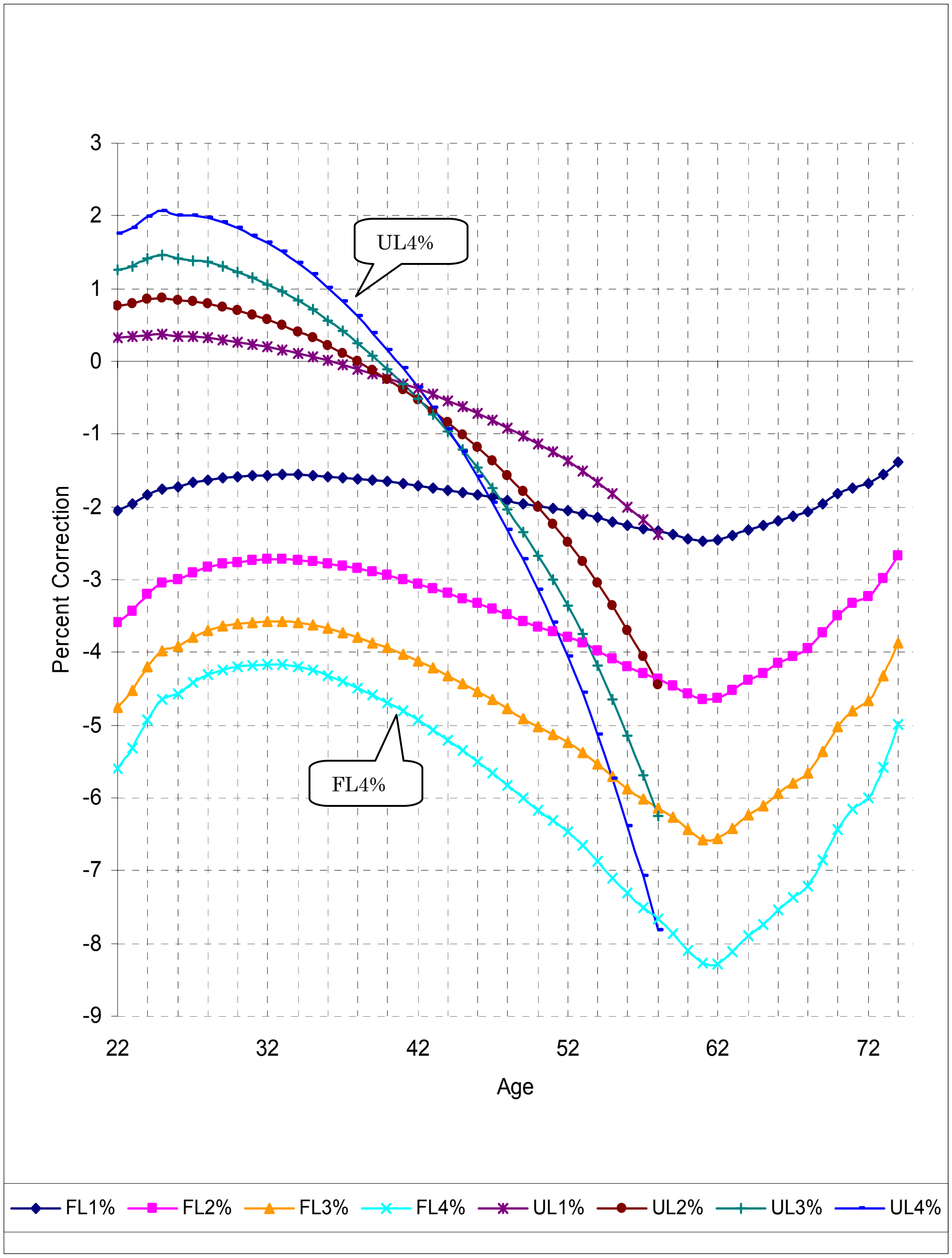


Figure 5. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Men with a Bachelor's Degree But No Graduate Degree

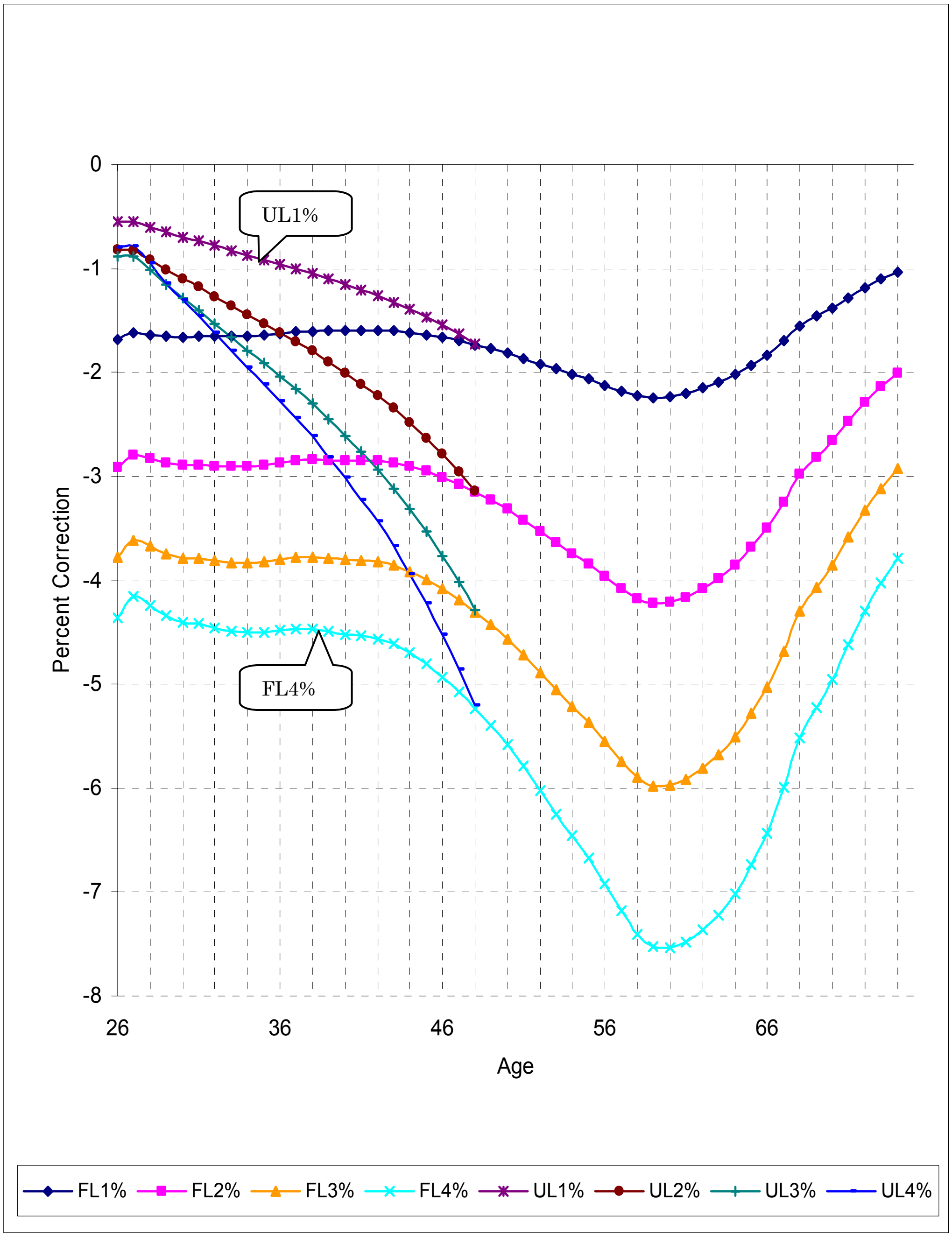


Figure 6. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Men with a Graduate Degree

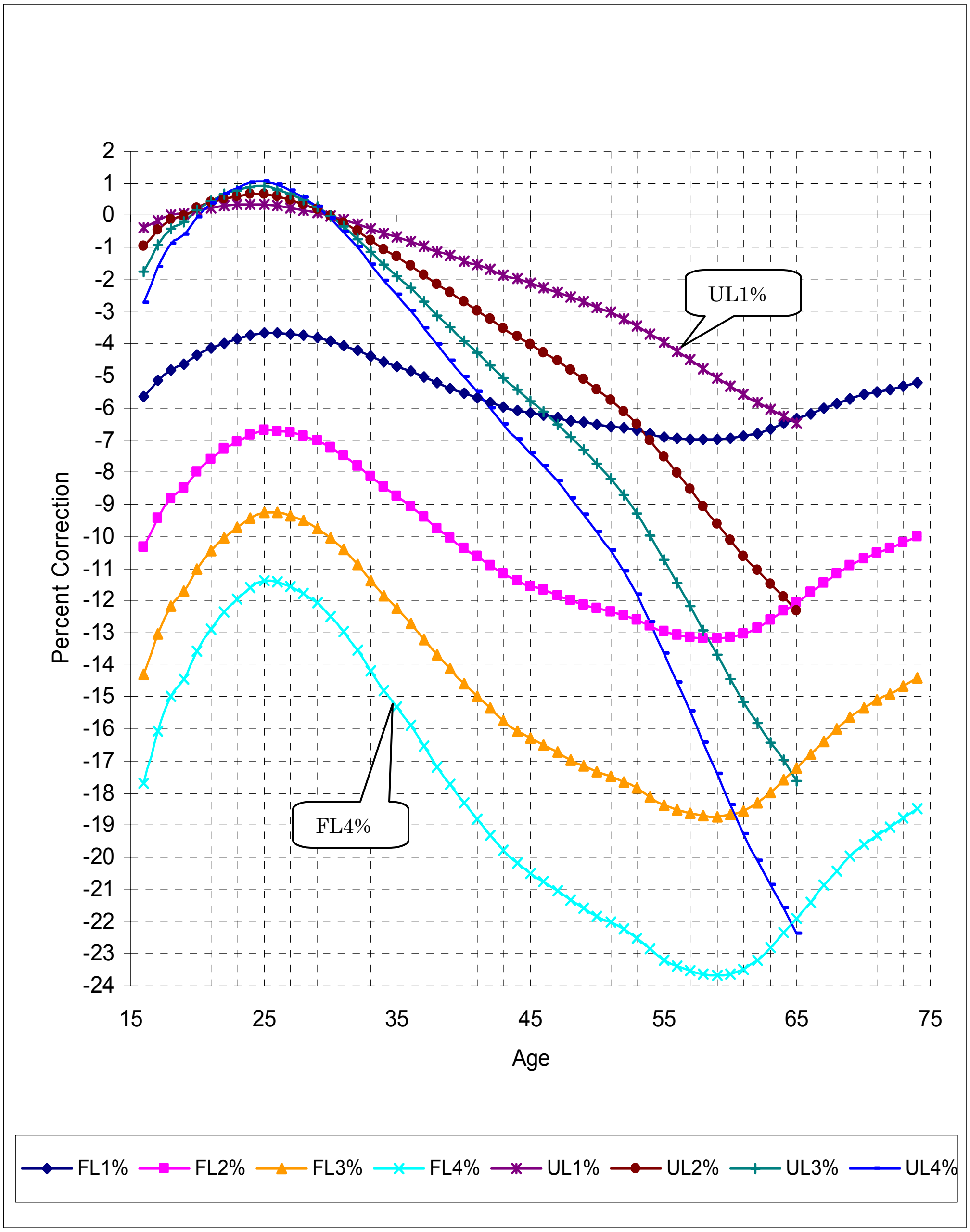


Figure 7. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Men, Regardless of Education

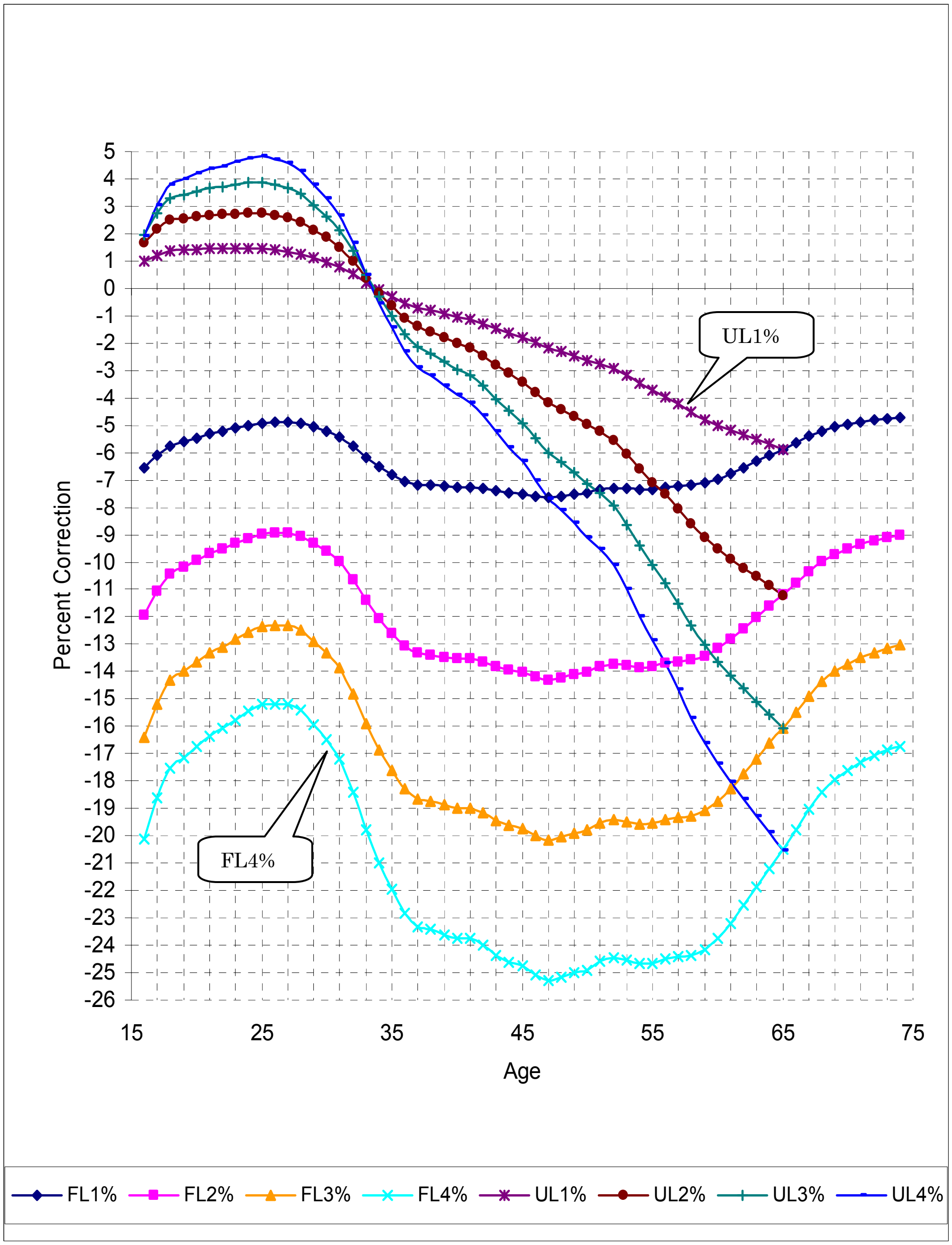


Figure 8. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Men with Less Than a High School Diploma

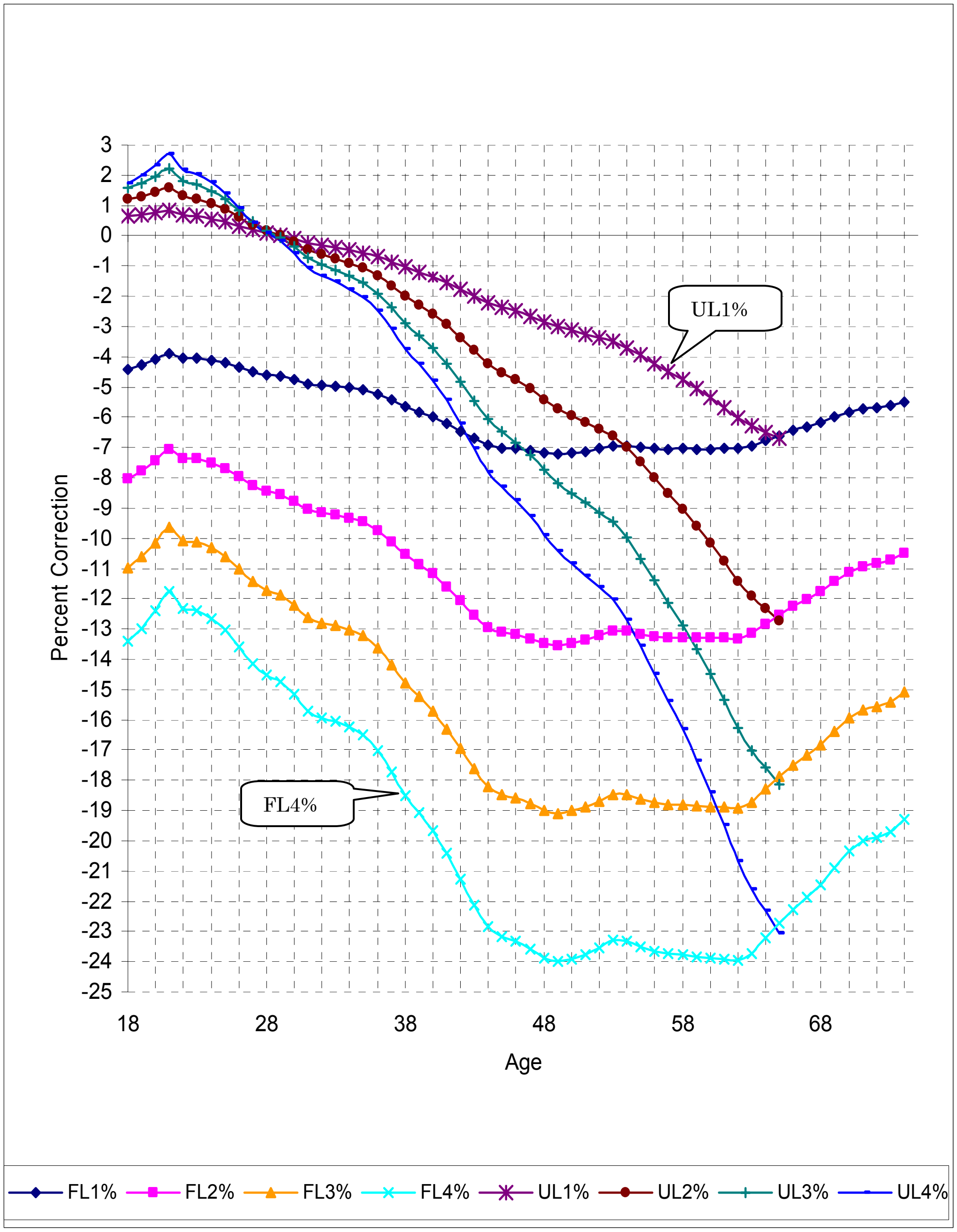


Figure 9. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Men with a High School Diploma Only

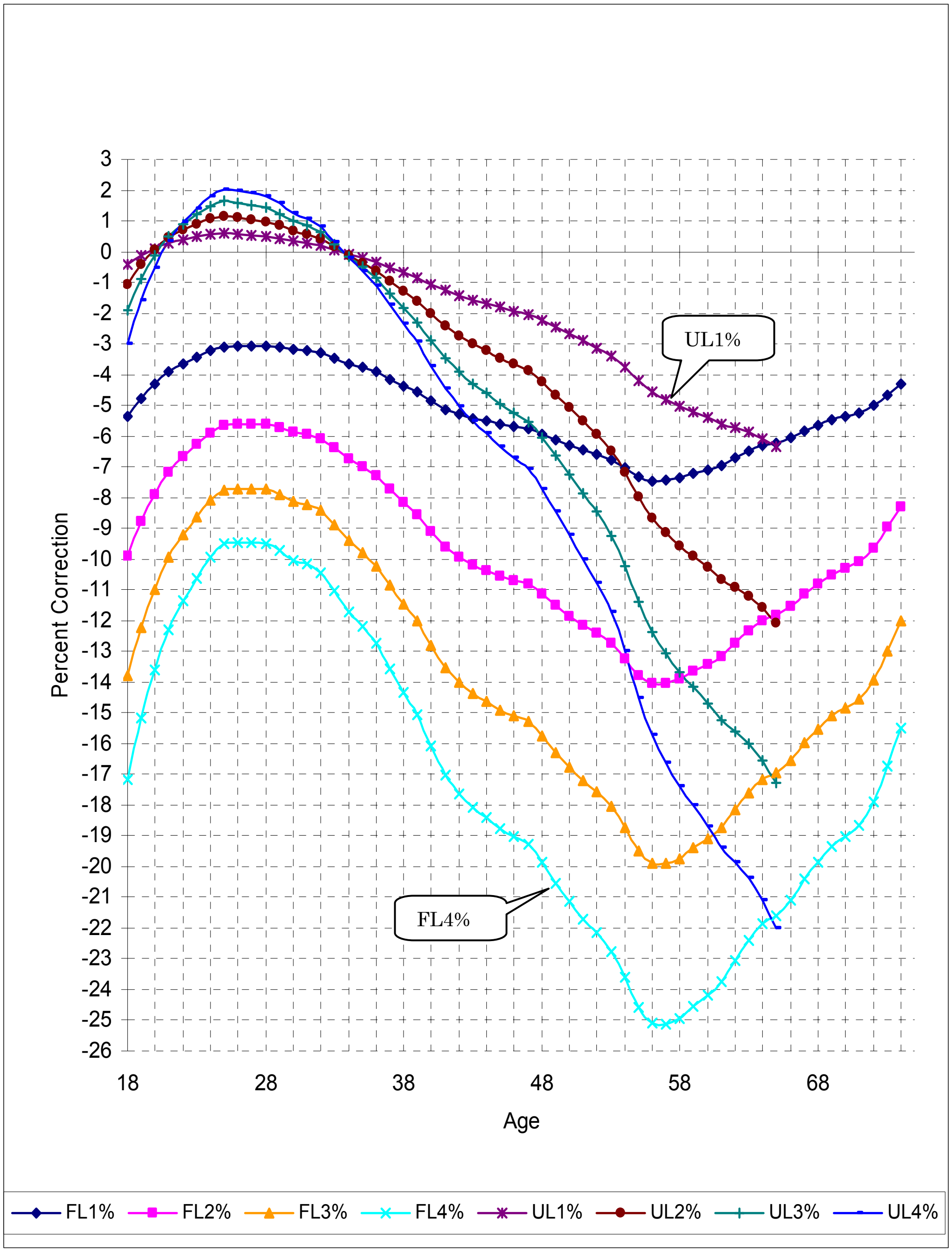


Figure 10. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Men with Some College But No Bachelor's Degree

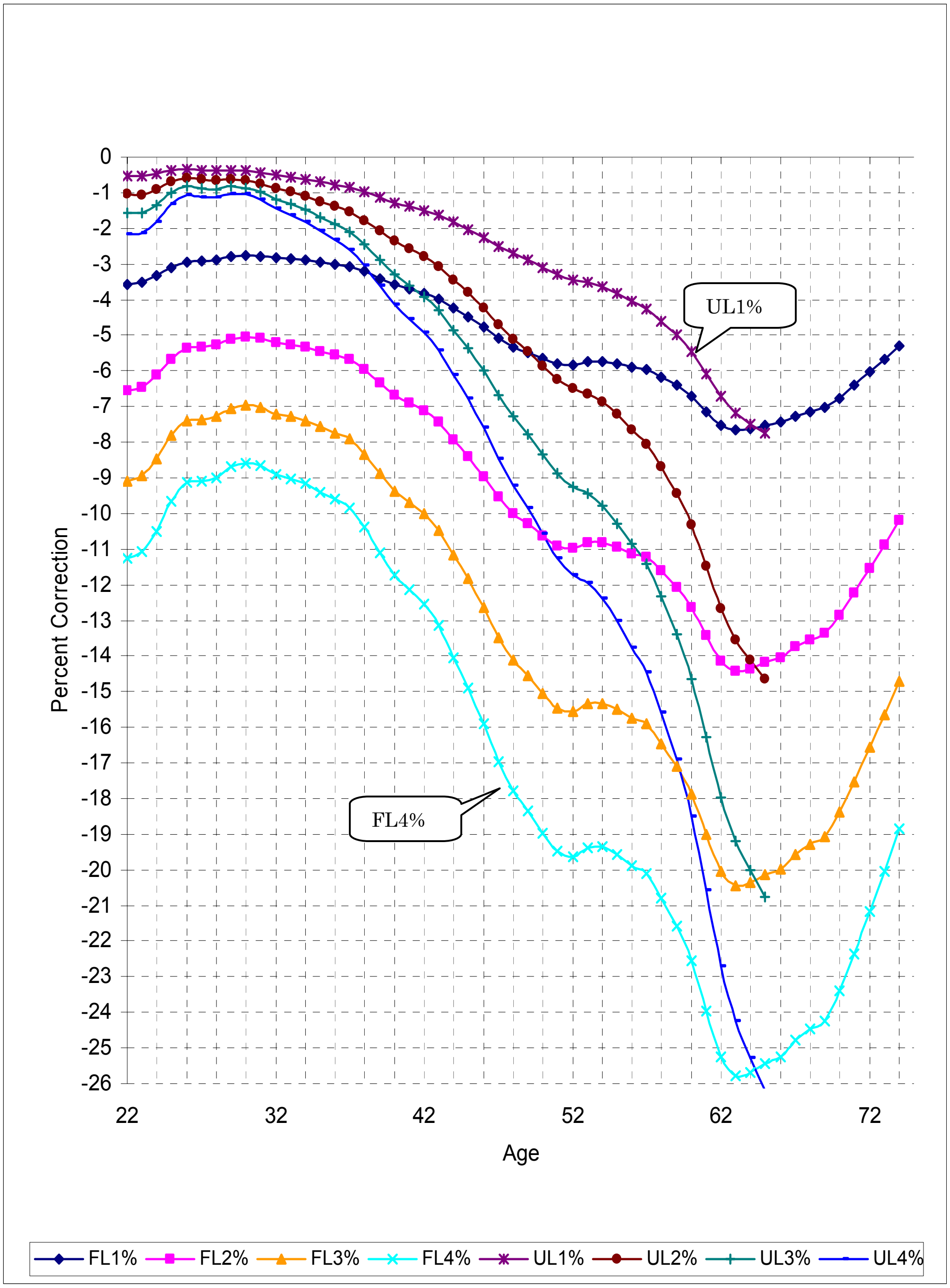


Figure 11. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Men with a Bachelor's Degree But No Graduate Degree

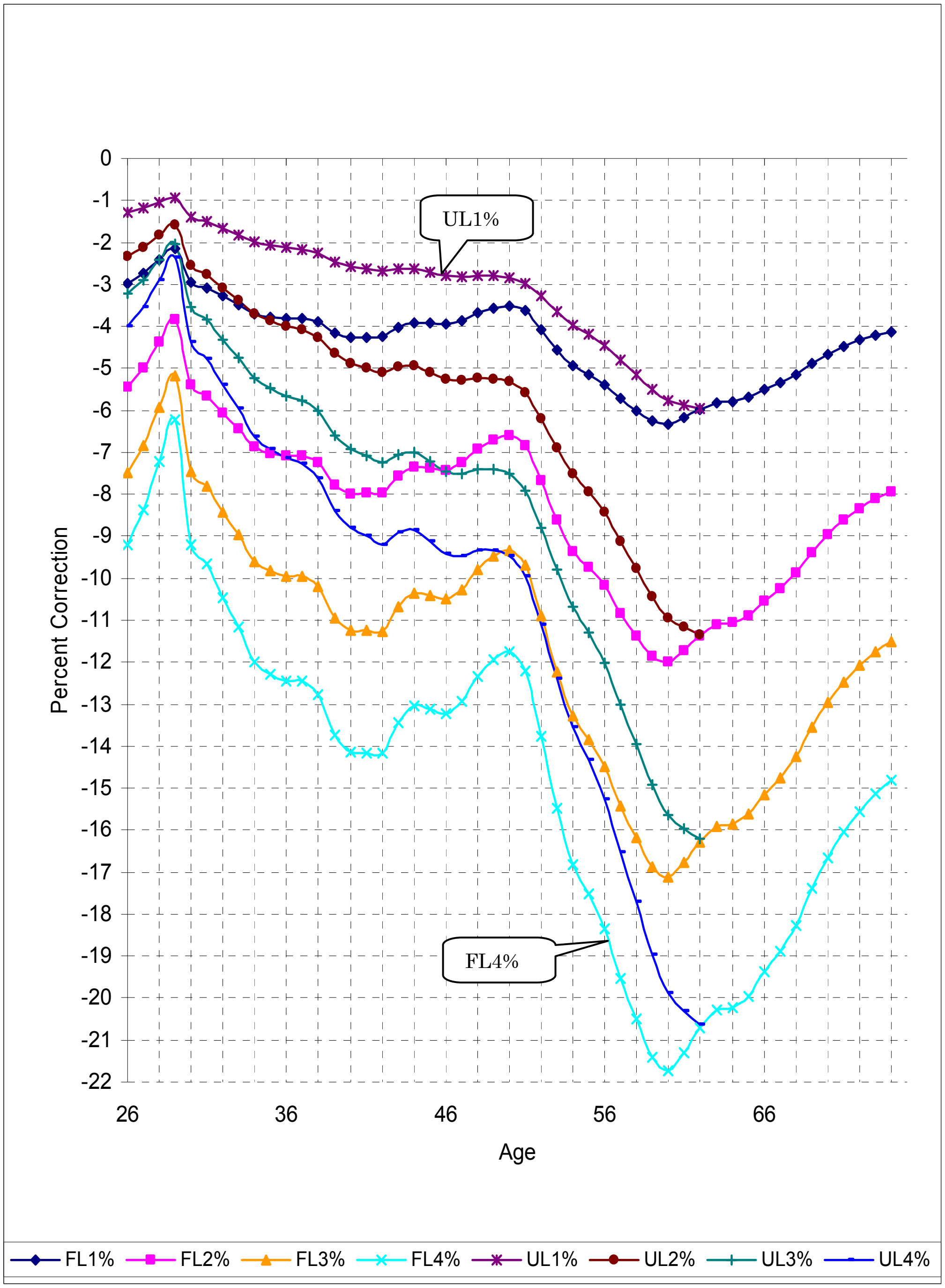


Figure 12. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Men with a Graduate Degree

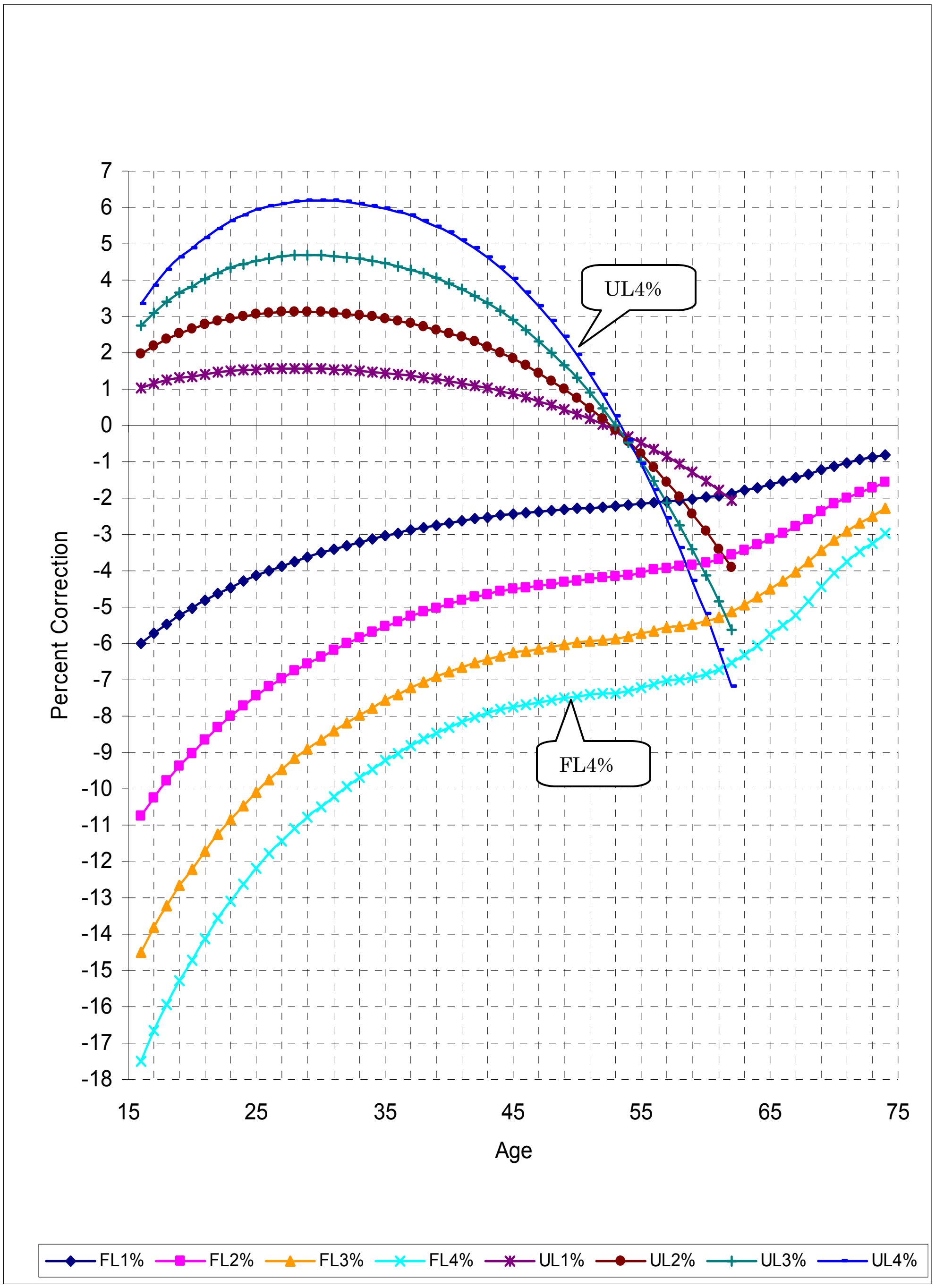


Figure 13. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Women, Regardless of Education

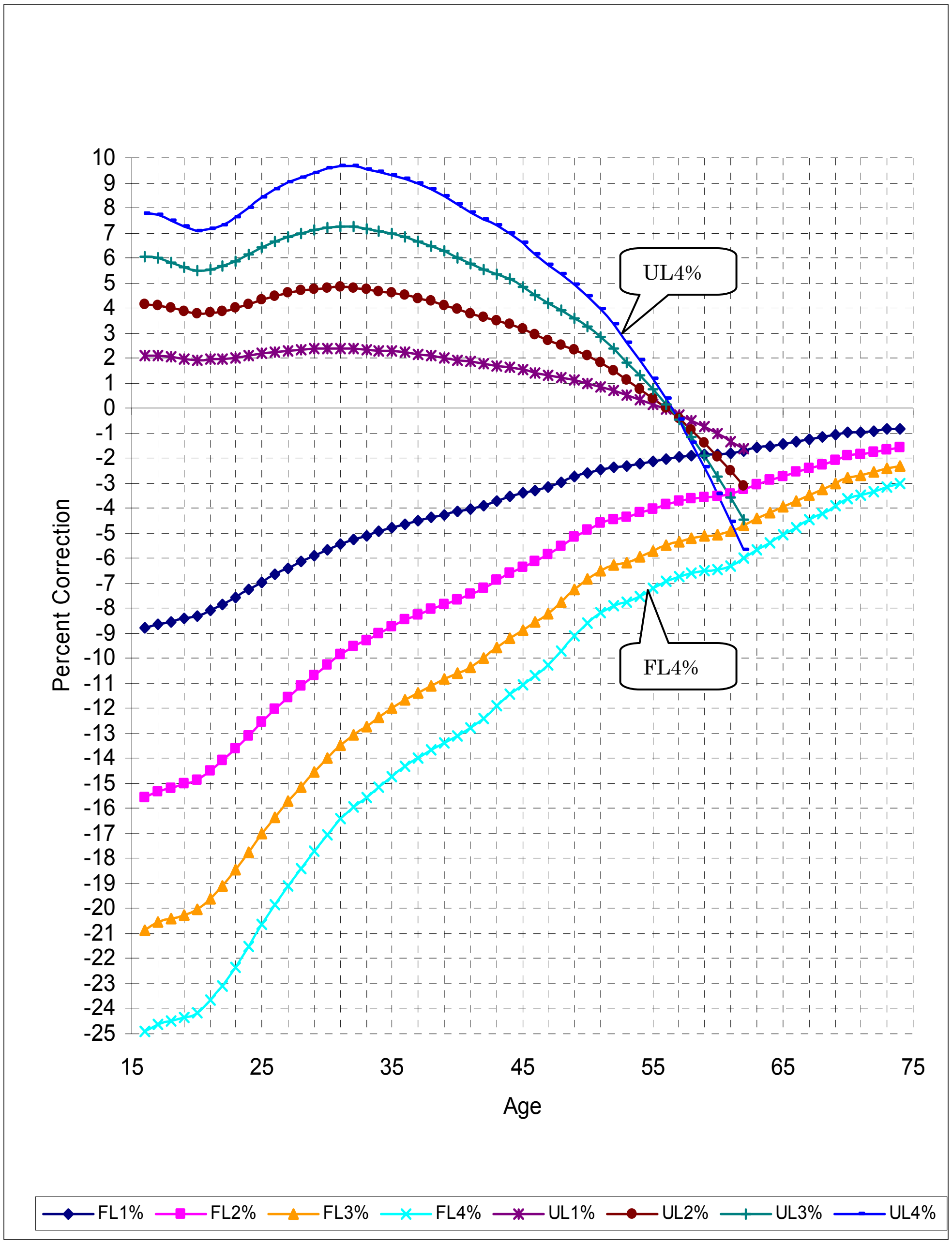


Figure 14. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Women with Less Than a High School Diploma

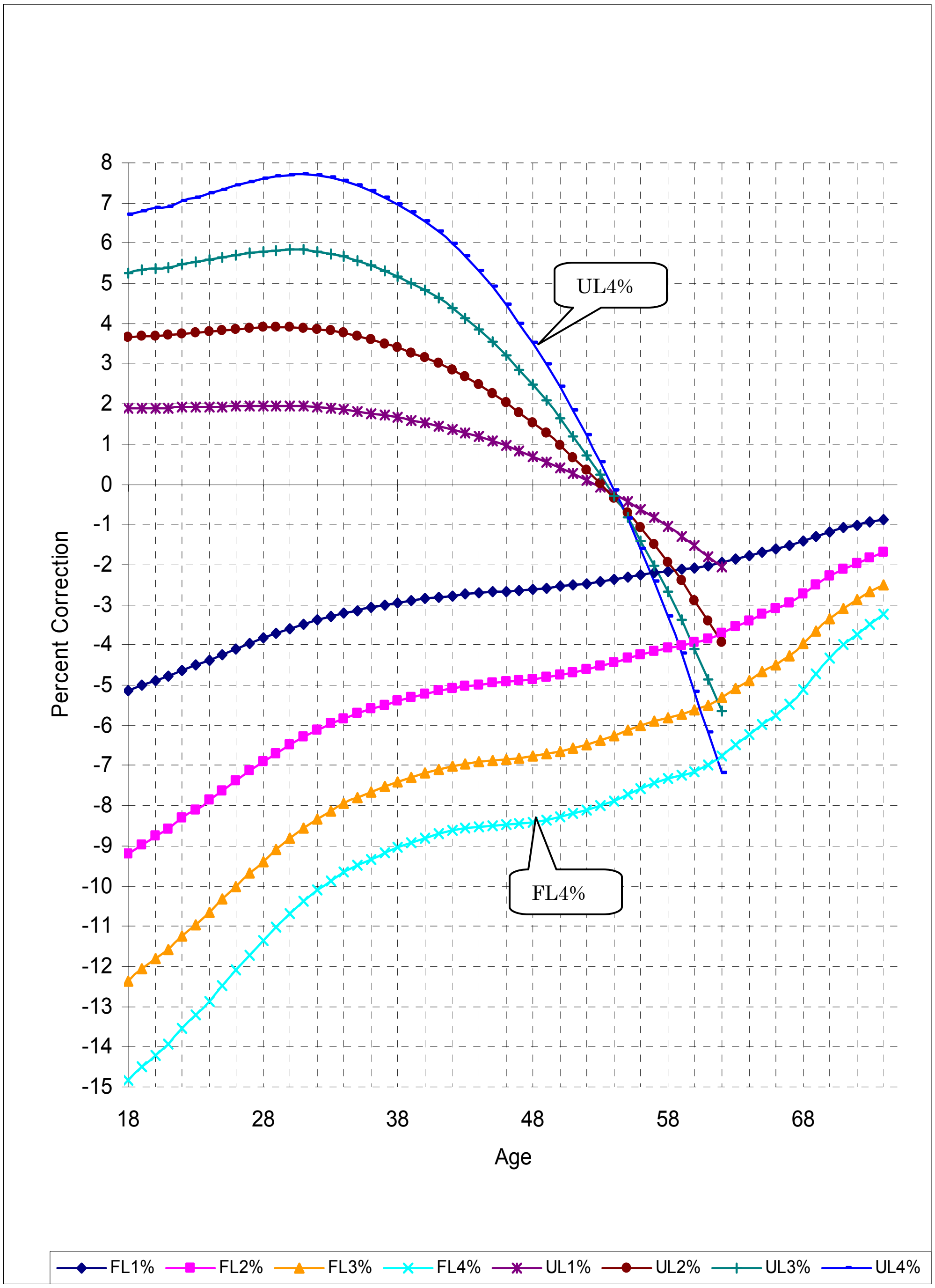


Figure 15. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Women with a High School Diploma Only

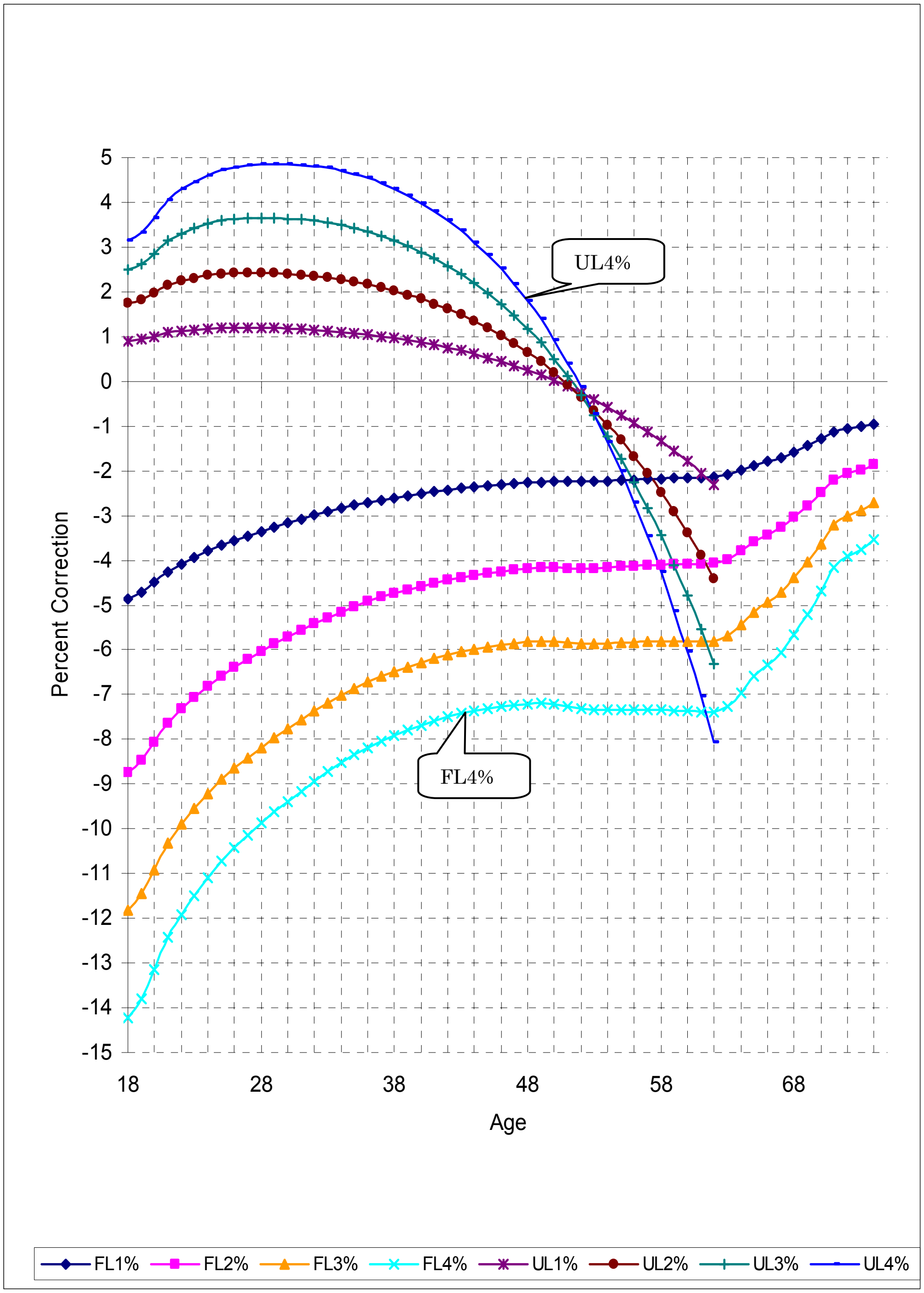


Figure 16. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Women with Some College But No Bachelor's Degree

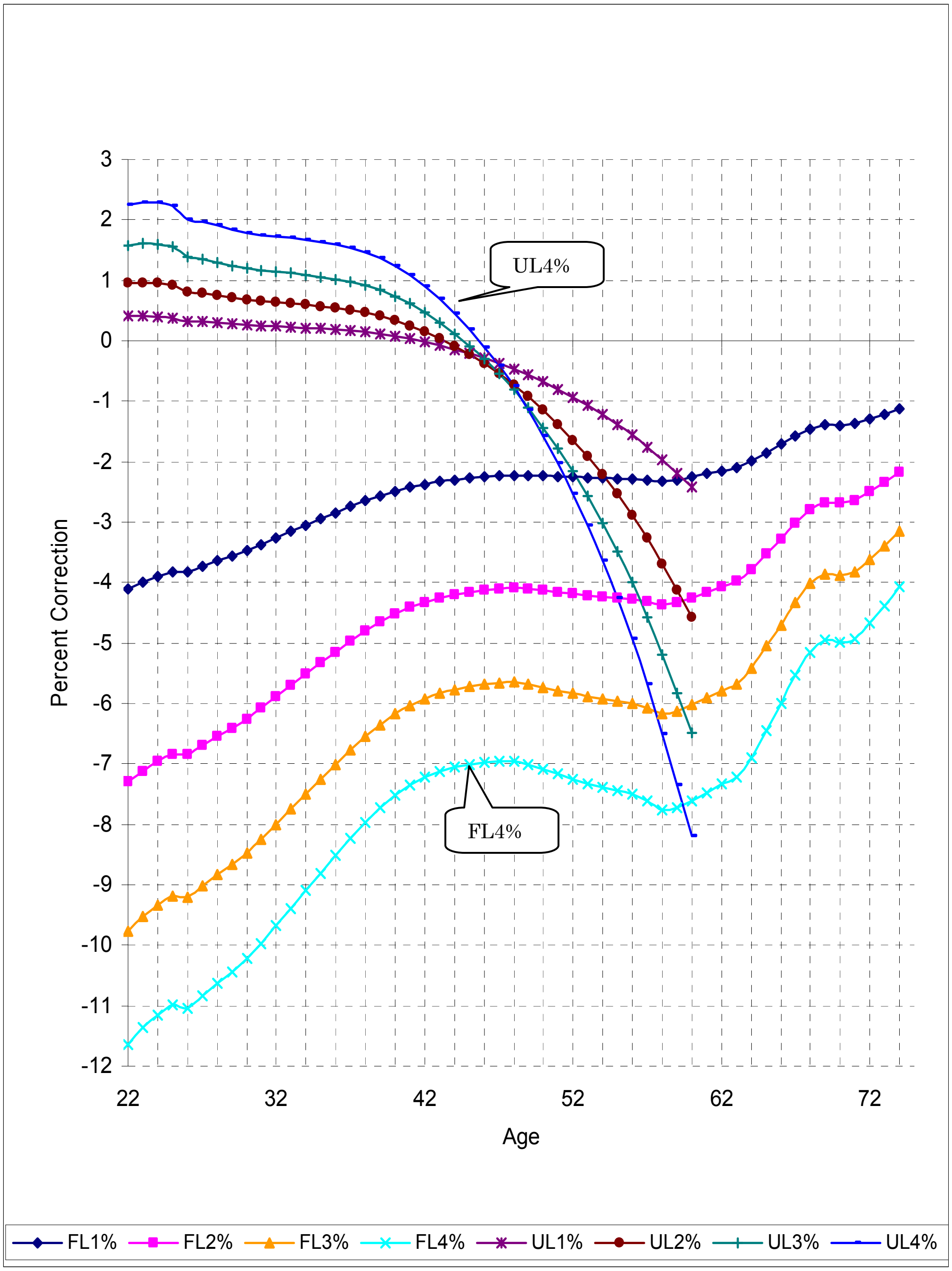


Figure 17. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Women with a Bachelor's Degree But No Graduate Degree

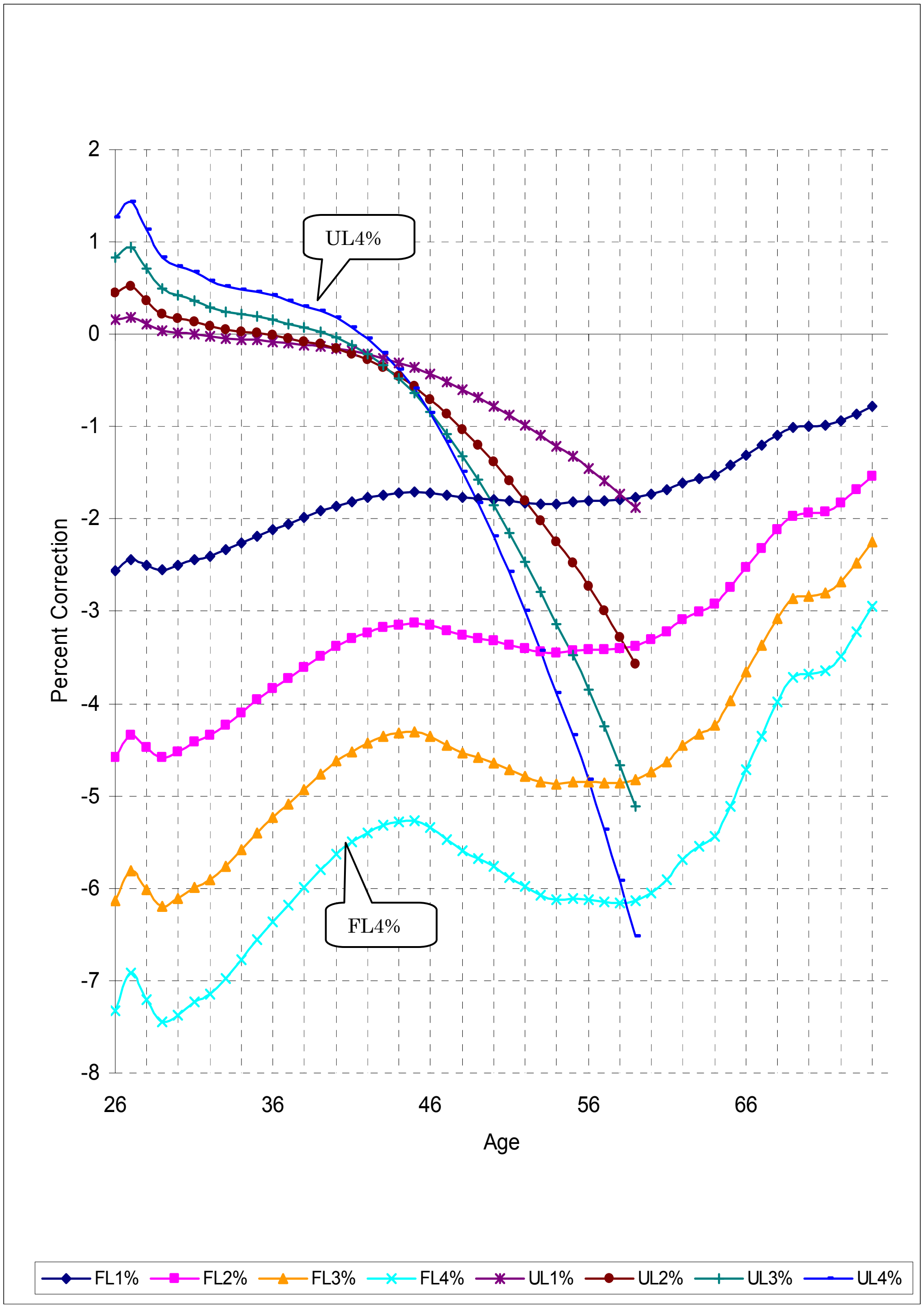


Figure 18. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Active Women with a Graduate Degree

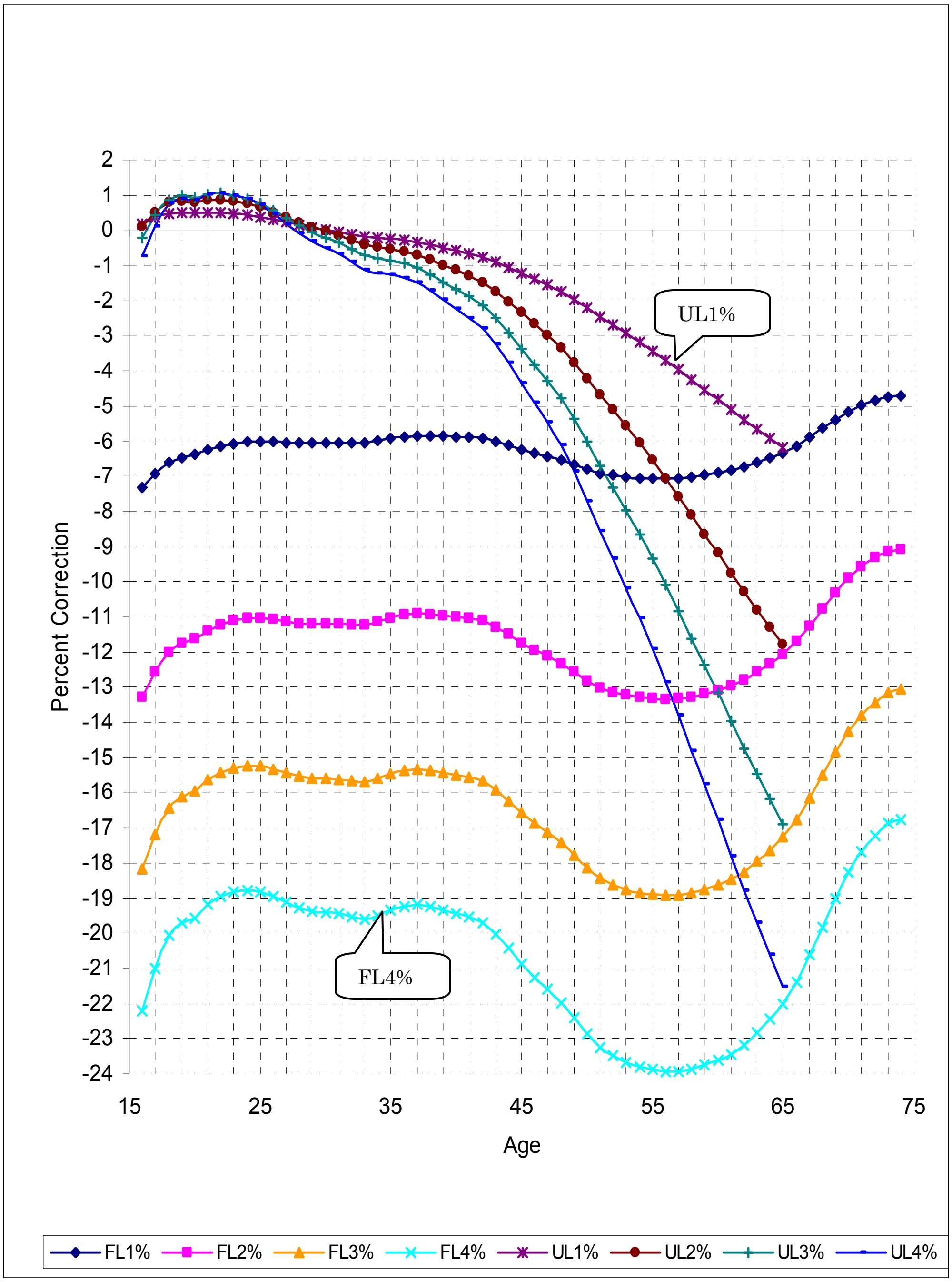


Figure 19. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Women, Regardless of Education

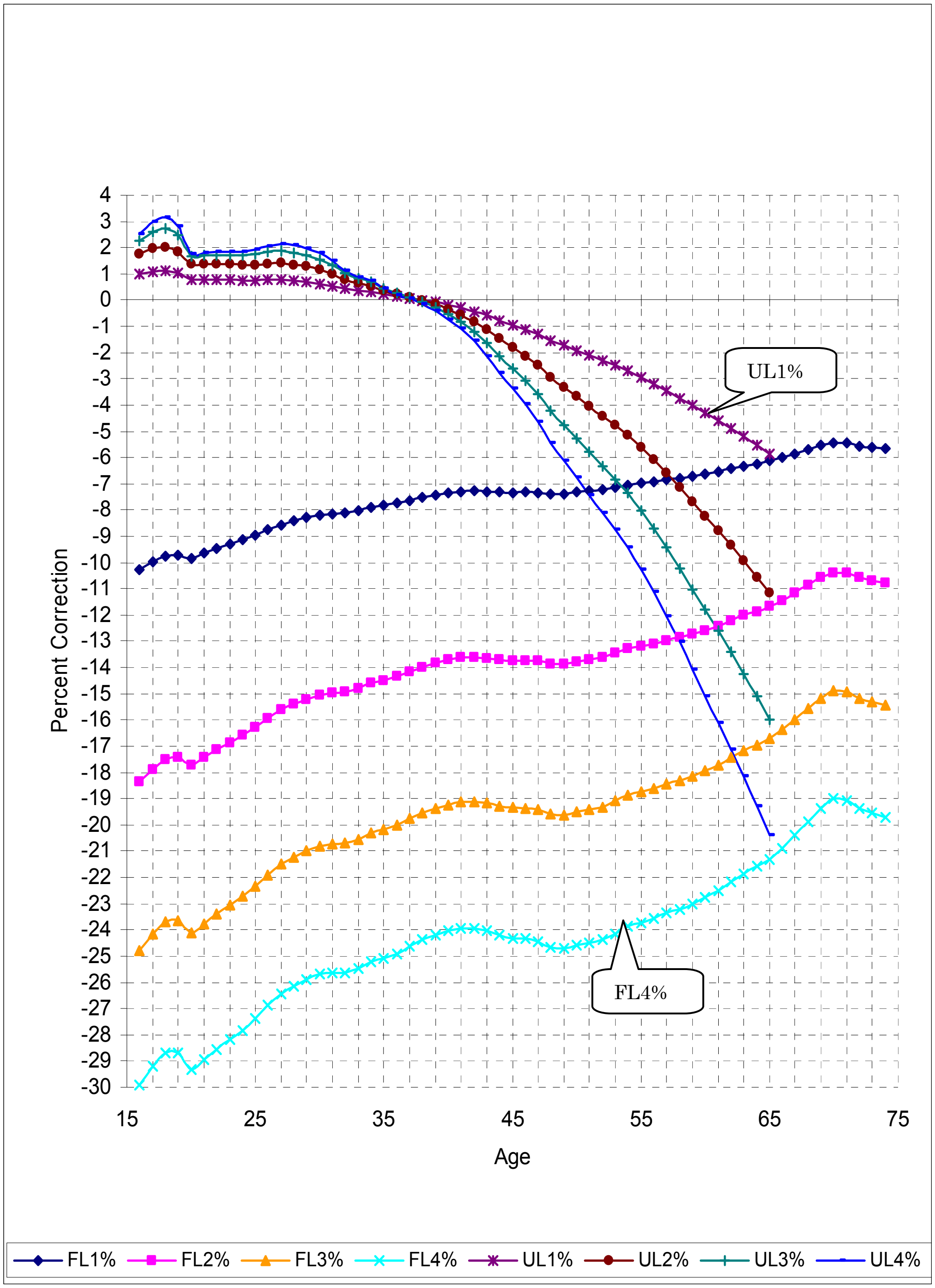


Figure 20. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Women with Less Than a High School Diploma

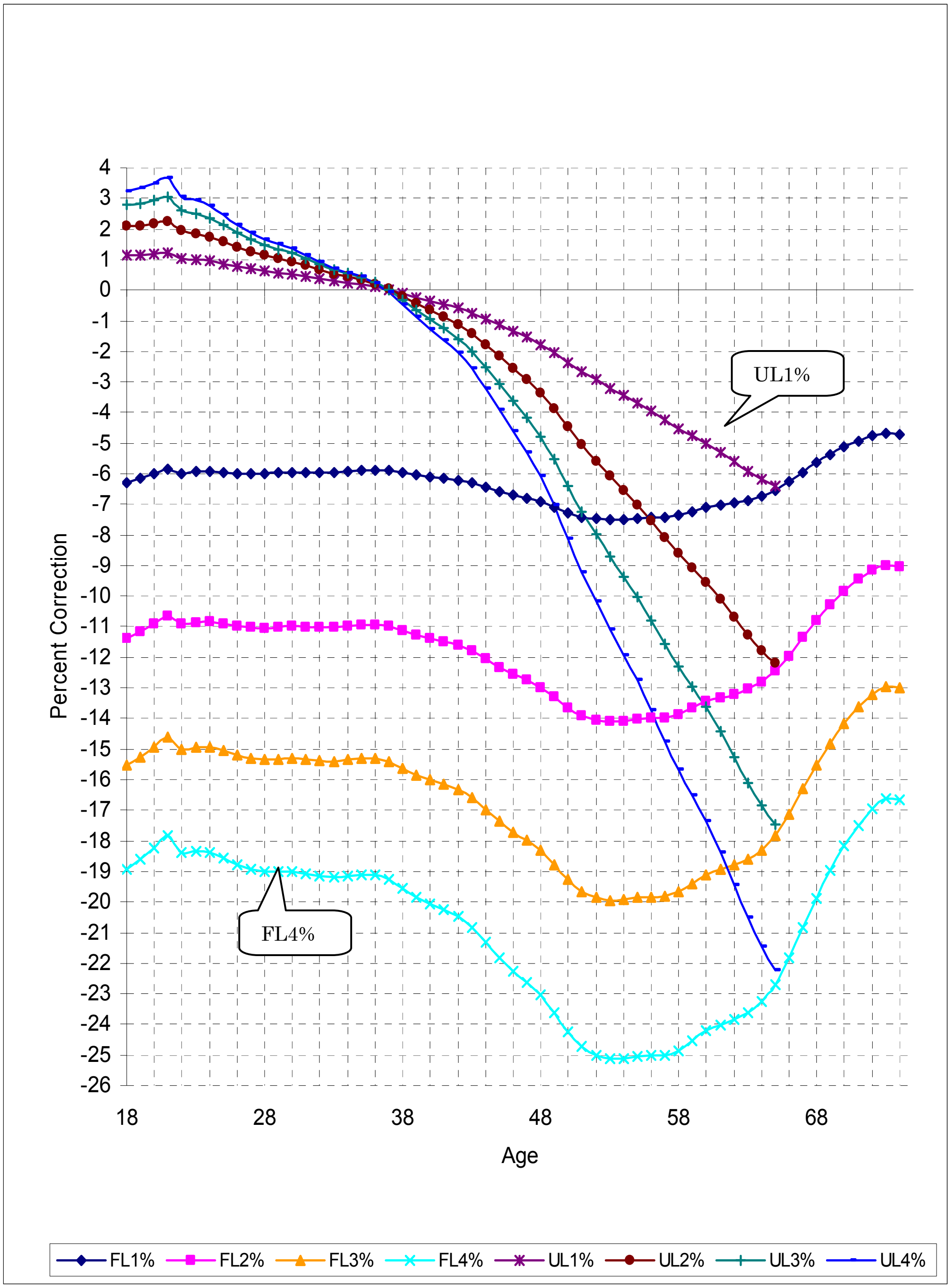


Figure 21. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Women with a High School Diploma Only

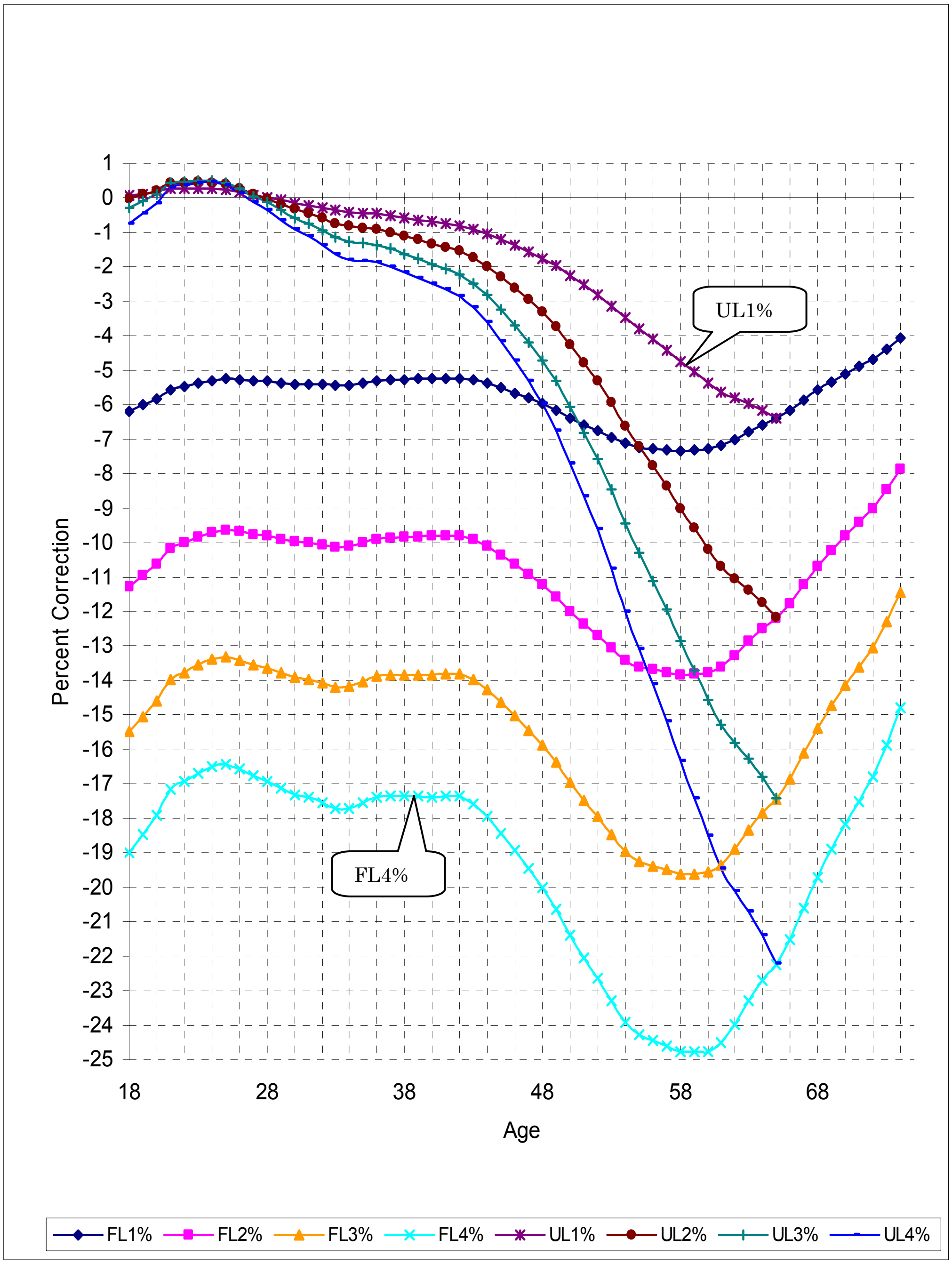


Figure 22. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Women with Some College But No Bachelor's Degree

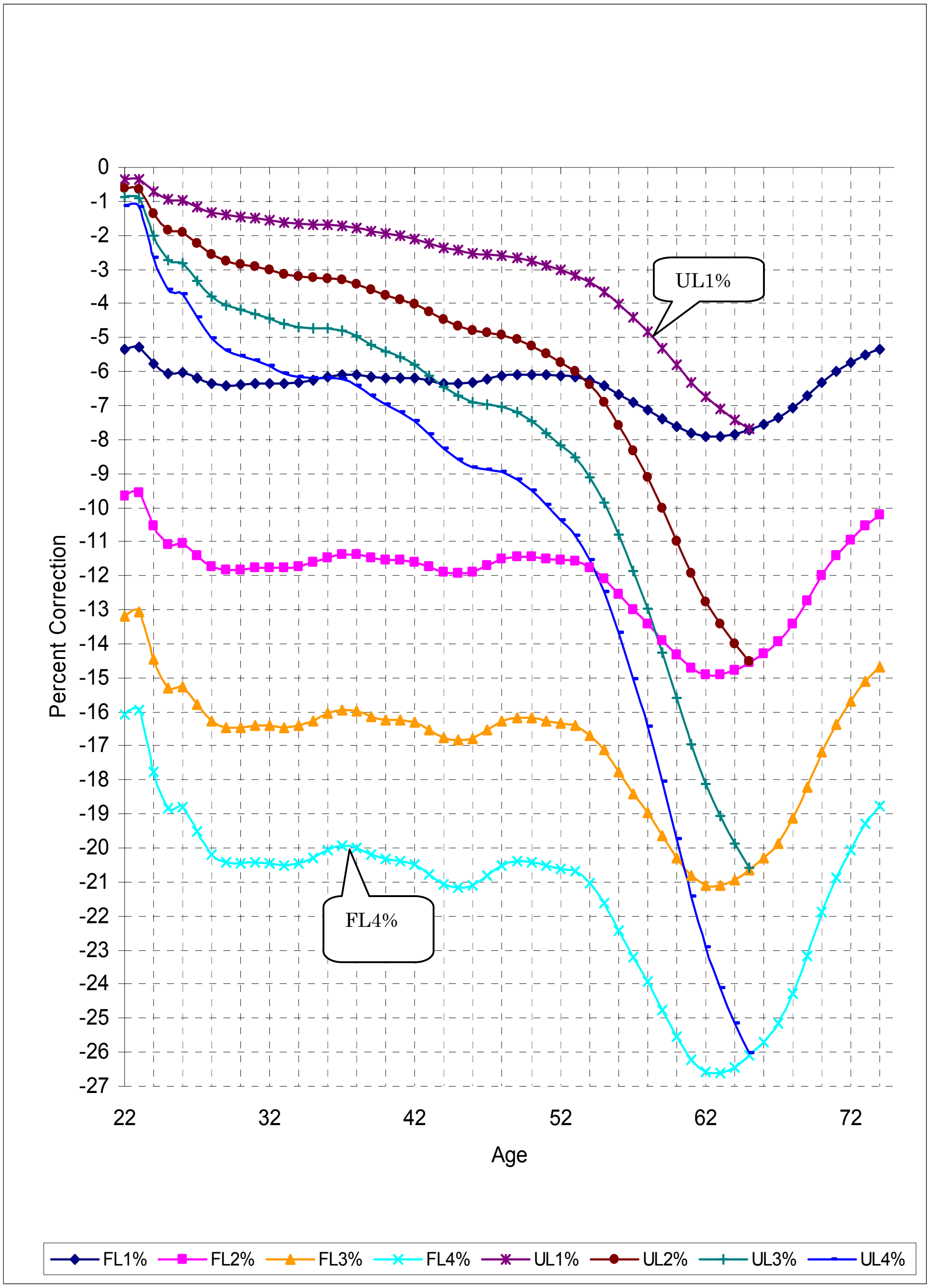


Figure 23. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Women with a Bachelor's Degree But No Graduate Degree

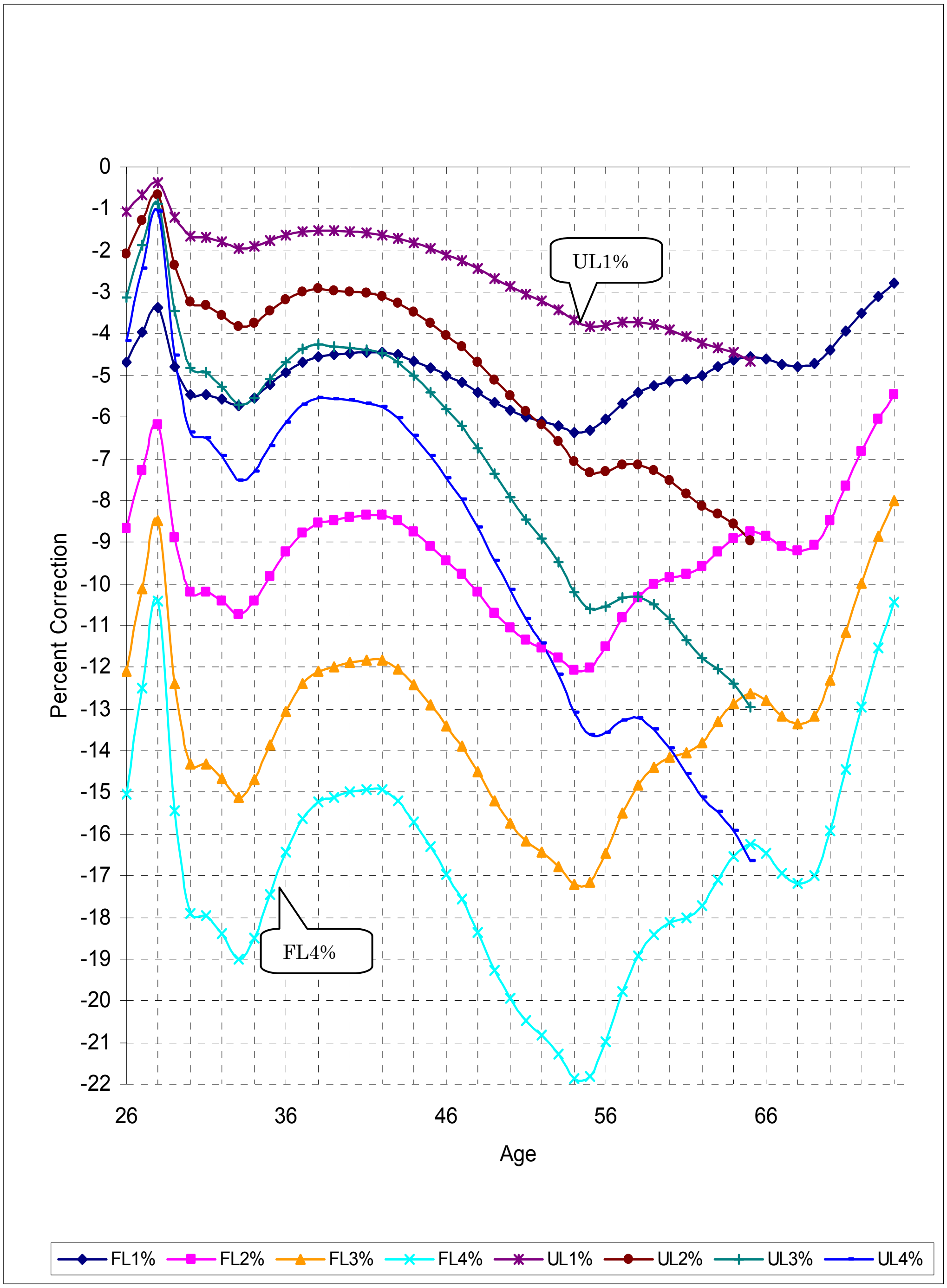


Figure 24. Percent Correction with Uniform Loading (UL) to Age 66 and Front Loading (FL) for Inactive Women with a Graduate Degree

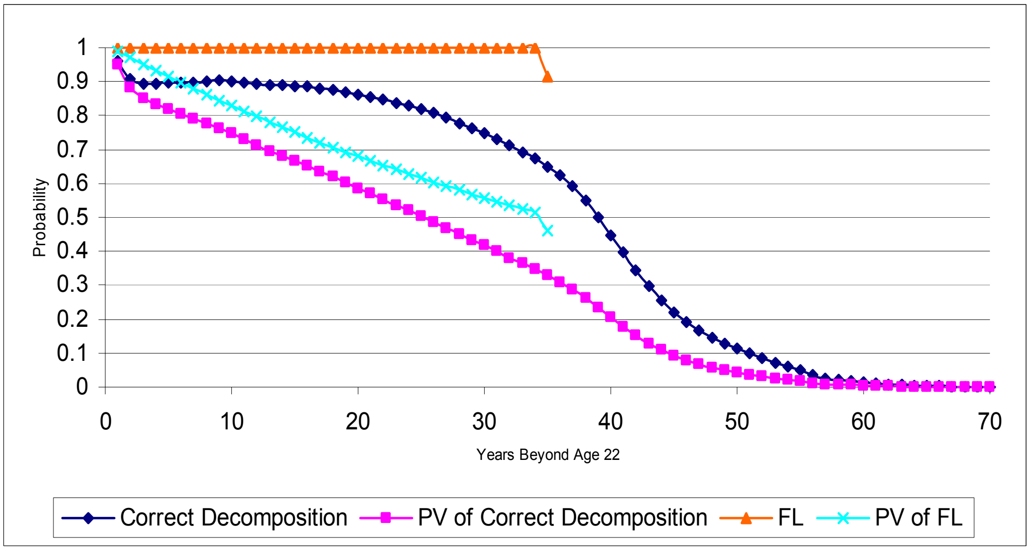


Figure 25. Correct Decomposition, PV of Correct Decomposition, Front Loading and PV of Front Loading at 2% NDR for Active Males Age 22 with High School Only

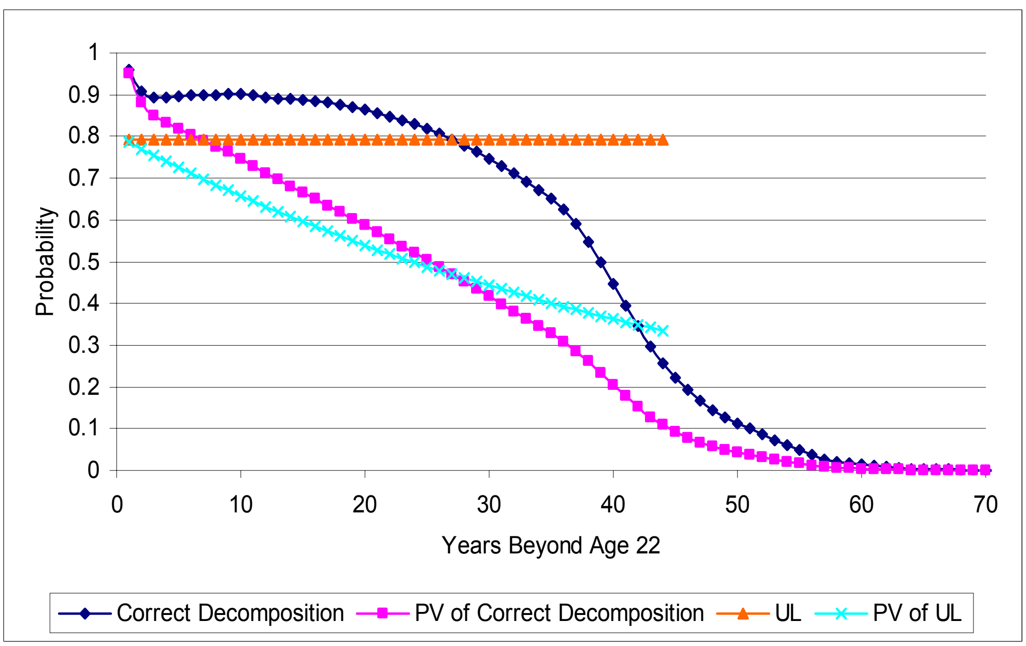


Figure 26. Correct Decomposition, PV of Correct Decomposition, Uniform Loading and PV of Uniform Loading at 2% NDR for Active Males Age 22 with High School Only

## IV. More Refined Treatment: Time from Personal Injury Date to Trial Date

The decomposition discussed so far is theoretically correct, in the sense of corresponding to the expectation of years of future activity accumulated over future intervals, conditional on information on the accident date. In two situations, this is clearly appropriate: 1. in wrongful death cases; 2. in personal injury cases where footnote 22 of *Jones & Laughlin Steel v. Pfeifer* (1983) is followed. (This footnote, widely ignored, suggests that expected present value calculations be made as of the injury date and to which, under federal law, prejudgment interest will be added).

When time passes between the injury date and the trial date in a personal injury case, and assuming that the plaintiff does not die during this period from other and independent causes, we may ask how the calculations should be altered (Townsend, 1997). Since the person has survived, and the worklife calculation incorporated some probability of dying over the age $x$ to age $x+t$ interval, we might wish to incorporate this information into our analysis. As of date $x+t$ , we have a different information set than was present as of the trial date, at least in this regard. Two related questions concern how we allocate the $t$ years to trial and the probabilities of activity and inactivity at the trial date $x+t$. Any forensic economist using the Markov model needs to deal, implicitly or explicitly, with these issues.

One approach is simply to assume that the plaintiff would have participated with 100% probability over this interval, and to re-calculate the worklife expectancy as of the trial date. A second approach would make the same assumption of 100% participation, but would subtract $t$ years from the age $x$ worklife expectancy, and employ front loading or uniform loading to the remainder. A third variant would proceed as in the second, but would weight the remaining years proportionally to something more readily measurable, like survival probability. None of these approaches explicitly corrects for the mortality gain experienced over the $t$ pre-trial years. We sketch the correct approach to this part of the problem, and conclude with additional complications.

Consider the modification of the above transition matrix:

$$\text{(12)} \qquad \mathrm{P}_{\mathrm{x}}^{*} = \frac{1}{{}^{a}p_x^{i} + {}^{a}p_x^{a}} \begin{pmatrix} {}^{a}p_x^{a} & {}^{i}p_x^{a} \\ {}^{a}p_x^{i} & {}^{i}p_x^{i} \end{pmatrix} \equiv \begin{pmatrix} {}^{A}p_x^{A} & {}^{I}p_x^{A} \\ {}^{A}p_x^{I} & {}^{I}p_x^{I} \end{pmatrix},$$

the transition matrix at age $x$ which ignores mortality. This may be used in place of $P_x$ in the previous development over the interval $(x,x+t)$ to appropriately modify the formulae. For example, from

$$\text{(13)} \qquad {}_{j}\Pi_x^{*} \equiv P_{x+j-1}^{*} P_{x+j-2}^{*} \cdots P_x^{*}$$

we modify and provide the explicit representation for the parts of years in the labor force in $(x,x+t)$; $j$ is set to 0, 1, …,[$t$] in

$$(14) \qquad {}_jE_x^* \equiv \begin{pmatrix} {}^a_je_x^{*a} & {}^i_je_x^{*a} \\ {}^a_je_x^{*i} & {}^i_je_x^{*i} \end{pmatrix} = \frac{1}{2}\,{}_j\Pi_x^* + \frac{1}{2}\,{}_{j+1}\Pi_x^* \equiv \frac{1}{2}P^*_{x+j-1}P^*_{x+j-2}\cdots P^*_x + \frac{1}{2}P^*_{x+j}P^*_{x+j-1}\cdots P^*_x$$

Since the years to trial will generally not be an integer, the last term will be

$$(15) \qquad (t-[t])\,{}_{[t]}E_x^* .$$

Using (13) with $j$=[$t$] and $j$=[$t$]+1 also provides (via interpolation) probabilities that the plaintiff would be found in the active and inactive states as of the trial date. These probabilities are read from the first column of the matrix

$$(16) \qquad \begin{pmatrix} {}^a_t\pi_x^{*a} & {}^i_t\pi_x^{*a} \\ {}^a_t\pi_x^{*i} & {}^i_t\pi_x^{*i} \end{pmatrix} = \{1-(t-[t])\}P^*_{x+[t]-1}P^*_{x+[t]-2}\cdots P^*_x + (t-[t])P^*_{x+[t]}P^*_{x+[t]-1}\cdots P^*_x$$

From this calculation, ${}^a_t\pi_x^{*a}$ and ${}^a_t\pi_x^{*i}$ are multiplied by the previously derived mortality incorporated in the ${}_jE_{x+t}$ participation probabilities displayed in (3).

There may be more information on which to condition. If plaintiff's health remained good over the pre-trial period, his job did not vanish, other determinants of labor supplied remained unchanged, it may be reasonable to argue that plaintiff would have remained in the labor force over the entire period. This may provide a rationale for selection of the trial date as a date on which to calculate worklife expectancy. In this case, the complications of the equations (12)–(16) may be ignored, and the third justification for the previous analysis has emerged.

It may well be that the last few equations constitute a search for the "delusive exactness" against which *Jones & Laughlin Steel v. Pfeifer* warned at footnote 33. While the corrections may indeed be small, it is good to have them in the literature; we have never seen this issue dealt with either theoretically or practically in forensic economics.

## V. Conclusion and the Past and Future Roles of Worklife Expectancy

Like life expectancy, worklife expectancy is an index number which stands in for an array of numbers that, if available, would render the calculations to which it is put more precise. Both expectancies are means of random variables, and the distributions of the underlying random variables have been quantified in recent years. Worklife expectancy has name and judicial recognition. Publicly available tables of worklife expectancy serve to limit the expert abuse of using, in many situations, without adjustment, naïve over-estimates such as 65-age, 67-age, or, worse, life expectancy, in the calculations. The present paper permits a quick and reasonably accurate assessment of biases commonly observed. Further, the nomograms may be used either directly, when applied to a year-by-year front loaded or uniformly loaded chart, or in conjunction with the selection of other assumptions. For example, if the front-loading bias is 3%

to 4%, the expert might ignore the incorporation of plaintiff's social security in the fringe benefits percentage in some circumstances. Notwithstanding these observations, a publicly available Excel macro or subroutine performing the exact decomposition incorporating the extensions of this paper would undoubtedly be welcomed by the forensic economics community, although it would require the display of many rows of near zero worklife probabilities between ages 75 and 90 or later, perhaps unnecessarily extending and cluttering charts.

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