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Present Value Functions and Recursions

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I. Introduction

Using a Markov-induced decomposition of time in the labor force, we compute means, standard deviations, and other distributional characteristics of the present value of years of labor force activity. We also provide bootstrap estimates of the mean present value and the corresponding standard deviations of sample means. This paper combines years of future labor force activity, decomposed with a Markov process, with discounting that activity to the present, whereas previous literature has analyzed only undiscounted years of activity.

A worker at exact age *x*, given activity status, sex and education, receives $1 for each future year in the labor force. The present value of this stream, which is assumed to grow at rate *g* and to be discounted at the interest rate *r*, is a random variable, since it depends on future labor market realizations and mortality experience. Letting *NDR* be the net discount rate, $(r-g)/(1+g)$, we define this random variable to be $PVA(a,x,NDR)$ when the person begins active and $PVA(i,x,NDR)$ when commencing inactive. We assume that future labor force activity status follows the usual Markov or increment-decrement model. In arranging these random variables into the row random vector $PVA(x,NDR)' \equiv (PVA(a,x,NDR), PVA(i,x,NDR))$, we provide a recursion for its probability mass function (often abbreviated "pmf" below), which we show to be computationally intractable. Next, we indicate how we may nevertheless estimate the probability mass function. We then provide a computationally useful recursion for its expected value $E[PVA(x,NDR)']$. These expected values have been tabulated in the United Kingdom, where they are associated with the Ogden Tables. From another point of view, $E[PVA(x,NDR)']$ is a generalization of the worklife expectancy since $E[PVA(x,0)'] = \left({}^{a}e_x^a, {}^{i}e_x^a\right)$, the familiar worklife expectancies when $NDR = 0$. The same concepts apply to flows of $1 while in the inactive state, and our computationally feasible recursion covers it as well.

It is well known that a person's worklife expectancy does not provide enough information to accurately compute the associated present value, since worklife will on average be allocated over future years in a particular pattern dictated by the underlying Markov model. Skoog and Ciecka (2006) took up a graphic way to perform this allocation. Other papers (Skoog and Ciecka, 2001a, 2001b and 2002) pointed out the intrinsic variability of the random variables $YA_{x,a}$ and $YA_{x,i}$ (years of activity starting active and inactive) around their

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respective means. The present paper brings these two ideas together, adds an extension of a recursion and decomposition found in Skoog (2002), to address what must surely be among the most natural and interesting questions for forensic economists: (1) What does the mean of the present value random variable, correctly calculated with the Markov-induced decomposition, look like at various *NDR's*? (2) What is the size of the standard error of the estimated mean of the present value random variable at various *NDR's*? (3) What does the present value distribution look like at different net discount rates? While the so-called Ogden Tables have asked about the first and second questions in the context of British data and a single *NDR*, it will be helpful to have these tables calculated for American data based on labor force participation and various *NDR's*. The third question has not even been previously asked. This is surprising; in the age of *Daubert* we feel that displaying variation as standard errors is important. Perhaps the lack of attention to this question has occurred because of the technical difficulties highlighted in this paper. We suggest another reason, however. Many forensic economists have implicitly understood or assumed that what we carefully define as the present value random variable instead meant *expected* present value. Questions about variability have not arisen because the discourse simply did not allow it. Our intuition about the variability comes from observing that, when the *NDR* is zero, present value is years of activity, whose pmf has been tabulated. We expect the same variability for the present value random variable when the *NDR* exceeds zero, but pulled leftward and shrunk, due to discounting.

Beyond this introduction, the paper is organized as follows. Sections II and III contain notation to capture the probability structure and timing (of payments) convention used in the paper. Section IV deals with the intractable present value recursion for the present value function. On a first reading, this section may be skimmed to more quickly arrive at Sections V and VI containing mean recursions and tabular results, respectively. The paper concludes with a brief discussion of the Ogden Tables in Section VII and some final thoughts about the present value random variable and its expected value in Section VIII.

II. Notation Associated with the Probability Structure

We let Z_x denote the state of our worker (referred to in the masculine) at exact age *x*, so that $Z_x = a$ or $Z_x = i$. $TA - 1$ (where TA is "terminal age" or "truncation age") is an age at which all transitions occurring ½ of a year later are to the death state; this is illustrated as age 110 below. As in our previous work, we assume that transitions occur at midpoints, so that for the next half-year at least, he continues in the same state he occupied at age *x*. At age $x + ½$, the first transition occurs, which we formalize by defining the "increment" random variables $Z_{x,.5}$, depending on the state occupied at age *x*:

$$\text{(1a)} \qquad \{Z_{x,.5} \mid Z_x = a\} = \begin{cases} a \text{ with probability } {}^{a}p_x^{a} \\ i \text{ with probability } {}^{a}p_x^{i} \\ d \text{ with probability } {}^{a}p_x^{d} \end{cases}$$

$$\{Z_{x,.5} \mid Z_x = i\} = \begin{cases} a \text{ with probability } {}^{i}p_x^{a} \\ i \text{ with probability } {}^{i}p_x^{i} \\ d \text{ with probability } {}^{i}p_x^{d}\,. \end{cases}$$

In the same way,

(1b) $$\{Z_{x,1.5} \mid Z_{x,.5} = a\} = \begin{cases} a \text{ with probability } {}^{a}p_{x+1}^{a} \\ i \text{ with probability } {}^{a}p_{x+1}^{i} \\ d \text{ with probability } {}^{a}p_{x+1}^{d} \end{cases}$$

$$\{Z_{x,1.5} \mid Z_{x,.5} = i\} = \begin{cases} a \text{ with probability } {}^{i}p_{x+1}^{a} \\ i \text{ with probability } {}^{i}p_{x+1}^{i} \\ d \text{ with probability } {}^{i}p_{x+1}^{d} \end{cases} ,$$

and, generally,

(1c) $$\{Z_{x,j+.5} \mid Z_{x,j-.5} = a\} = \begin{cases} a \text{ with probability } {}^{a}p_{x+j}^{a} \\ i \text{ with probability } {}^{a}p_{x+j}^{i} \\ d \text{ with probability } {}^{a}p_{x+j}^{d} \end{cases} \qquad j = 1,2,\ldots,TA-x-1$$

(1d) $$\{Z_{x,j+.5} \mid Z_{x,j-.5} = i\} = \begin{cases} a \text{ with probability } {}^{i}p_{x+j}^{a} \\ i \text{ with probability } {}^{i}p_{x+j}^{i} \\ d \text{ with probability } {}^{i}p_{x+j}^{d} \end{cases} \qquad j = 1,2,\ldots,TA-x-1.$$

Let $t \geq 0$ refer to any (not necessarily integer or half integer) number of years beyond age *x*, and let $Z_x(t)$ be the random state occupied at this age, *x+t,* defined out of the increments in (1a)-(1d).

(2) $$Z_x(t) = \begin{cases} Z_x \text{ for } 0 \leq t < .5 \\ Z_{x,.5} \text{ for } .5 \leq t < 1.5 \\ \vdots \\ Z_{x,TA-x-1.5} \text{ for } TA - x - 1.5 \leq t < TA - x - .5 \\ Z_{x,TA-x-.5} = d \text{ for } TA - x - .5 \leq t \end{cases}$$

We construct a random variable which is left continuous and constant between half integers, and it changes on the half integer whenever the labor force status changes. The stochastic structure of $Z_x(t)$ depends on the initial state of Z_x and is induced by the probabilities $\{{}^{a}p_{x+j}^{a}\}_{j=0}^{j=TA-x-1}$, $\{{}^{i}p_{x+j}^{i}\}_{j=0}^{j=TA-x-1}$, $\{{}^{a}p_{x+j}^{d}\}_{j=0}^{j=TA-x-1}$. We can convert the function $Z_x(t)$, which takes on values of *a, i* and *d,* to a function which equals one for all time when in the active state, by use of the commonly used indicator function $I_{[Z_x(t)=a]}$, defined as: for any event *E*, $I_E = 1$ whenever the event *E* is true.

Our earlier work focused on YA_x, the years of additional labor force activity, starting at age *x* in state *m*. Evidently

$$(3) \qquad YA_{x,m} = \int_{t=0}^{t=TA-x-.5} I_{[Z_x(t)=a|Z_x=m]} dt .$$

If we indicate both age *x* and the initial state, *a* or *i*, in our notation, we can express this last equality as

$$(4a) \qquad YA_{x,a} = .5 + \sum_{j=.5}^{j=TA-x-.5} I_{[Z_{x,j}=a|Z_x=a]} \text{ and (4b) } YA_{x,i} = \sum_{j=.5}^{j=TA-x-.5} I_{[Z_{x,j}=a|Z_x=i]} .$$

Despite equation (3), this model is essentially discrete. A truly continuous model would posit instantaneous forces of increment, decrement and mortality, and payments would grow and be discounted with instantaneous forces. We take this model up elsewhere, but note here that in continuous time the "construction" of $Z_x(t)$ would not be required, simplifying the development above.

III. Notation Associated with Present Value

We need to specify when earnings are paid, how they are discounted, and how they grow. We are motivated by the fact that our transitions will be observed once per year, but compensation is paid much more frequently, rarely daily or annually, but more often bi-weekly, semi-monthly or monthly. In the case of monthly payments, the average of the payment points of $1/12$, $2/12$,..., $12/12$ of a year is 6.5 months or $13/24$ of a year. Consequently assuming one payment is made at the midpoint of $12/24$ introduces a very modest acceleration of $1/24$ of a year.

We allow for an age-earnings curve by including the sequence $\{ae_j\}$ for appropriate indices *j* relative to the base earnings level for age *x*, although much forensic practice, and our tabulated tables, will set $\{ae_j\}$ to unity. We follow standard forensic economic practice and assume that earnings grow at rate *g* and are discounted at rate *r*; these may be taken as either both nominal or both real. Since the effects of *r* and *g* separately enter the present value through the net discount rate, $NDR \equiv \frac{r-g}{1+g}$ we use the relations

$$(5a) \qquad \left(\frac{1+g}{1+r}\right) = \left(\frac{1}{1+NDR}\right), \text{ and the definitions}$$

$$(5b) \qquad \beta \equiv \left(\frac{1+g}{1+r}\right)^{.5} = \left(\frac{1}{1+NDR}\right)^{.5}, \text{ so that}$$

$$(5c) \qquad \beta^2 \equiv \left(\frac{1+g}{1+r}\right) = \left(\frac{1}{1+NDR}\right).$$

In light of the timing issue between payments and transitions, and consistent with the convention adopted in our previous work, we continue to assume that transitions take place at mid-period. Our first inclination is to consider the possibility of assuming that payments are made when transitions take place, *i.e.* at mid-periods. There is one asymmetry, namely, that we begin at an exact age, x, and the first transition is after ½ of a period, followed thereafter at periods one year apart. To fix ideas, assume the worker is active at x. Then \$.5 is earned between x and x+.5. Assume that the transition at x+.5 is active to active, so (i) \$.5 is earned for the activity in [x+.5, x+1) and (ii) \$.5 is earned in [x+1,x+1.5). We define two allocation conventions below: Convention A assumes that (i) \$.5 is paid at x+.5, ½ of a period before the mid-point of the interval [x+.5, x+1.5) while (ii) another \$.5 is paid at x+1.5, ½ of a period after the mid-point of the interval [x+.5, x+1.5). This convention splits payments for the interval into two pieces. Additionally, the transition which took place at age $x-.5$, and which is responsible for the activity in [x, x+.5), results in \$.5 being paid at x+.5, so all future work at x and beyond is paid in the future.

Convention B would put the entire payment at x+1, the midpoint of the [x+.5, x+1.5) interval. In this case, payments occur at exact ages, different points from transitions. Convention B requires that the payment for [x, x+.5) was made at x along with the payment for activity in [x-.5, x), and that these payments took place the instant before attaining age x, so that references to the present value of payments at exact age x start at age x+.5, with its associated one year interval [x+.5, x+1.5).

The present value random variables associated with Convention A are:

$$\text{(6a)}\quad \begin{aligned} PVA_{x,a} &= .5\left(\frac{1+g}{1+r}\right)^{.5} + \sum_{j=.5}^{j=TA-x-1.5} I_{[Z_{x,j}=a|Z_x=a]}\left\{ae_{j+1}.5\left(\frac{1+g}{1+r}\right)^{j+1} + ae_j.5\left(\frac{1+g}{1+r}\right)^{j}\right\} \quad \text{for } x < TA-1 \\ &= .5\beta + \sum_{j=.5}^{j=TA-x-1.5} I_{[Z_{x,j}=a|Z_x=a]}\left\{ae_{j+1}.5\left(\beta^2\right)^{j+1} + ae_j.5\left(\beta^2\right)^{j}\right\} \end{aligned}$$

$$PVA_{x,a} = .5\left(\frac{1+g}{1+r}\right)^{.5} = .5\beta \text{ for } x = TA-1$$

and

$$\text{(6b)}\quad \begin{aligned} PVA_{x,i} &= \sum_{j=.5}^{j=TA-x-1.5} I_{[Z_{x,j}=a|Z_x=i]}\left\{ae_{j+1}.5\left(\frac{1+g}{1+r}\right)^{j+1} + ae_j.5\left(\frac{1+g}{1+r}\right)^{j}\right\} \quad \text{for } x < TA-1 \\ &= \sum_{j=.5}^{j=TA-x-1.5} I_{[Z_{x,j}=a|Z_x=i]}\left\{ae_{j+1}.5\left(\beta^2\right)^{j+1} + ae_j.5\left(\beta^2\right)^{j}\right\} \end{aligned}$$

$$PVA_{x,i} = 0 \text{ for } x = TA-1 .$$

On Convention B, the expression on the right hand side does not depend on the initial state, although the distribution of $I_{[Z_{x,j}=a]}$ does, so that we have:

$$(6c)\ PVA_{x,a} = \sum_{j=1}^{j=TA-x-1} I_{[Z_{x,j}=a|Z_x=a]}\left\{ae_j\left(\frac{1+g}{1+r}\right)^j\right\} = \sum_{j=1}^{j=TA-x-1} I_{[Z_{x,j}=a|Z_x=a]}\left\{ae_j\left(\beta^2\right)^j\right\} \text{ for } x < TA-1$$

$$PVA_{x,a} = 0 \text{ for } x = TA-1$$

$$(6d)\ PVA_{x,i} = \sum_{j=1}^{j=TA-x-1} I_{[Z_{x,j}=a|Z_x=i]}\left\{ae_j\left(\frac{1+g}{1+r}\right)^j\right\} = \sum_{j=1}^{j=TA-x-1} I_{[Z_{x,j}=a|Z_x=i]}\left\{ae_j\left(\beta^2\right)^j\right\} \text{ for } x < TA-1$$

$$PVA_{x,i} = 0 \text{ for } x = TA-1\,.$$

The equations (6) display the source of the randomness in the present value random variable, and provide the suggestion for how we will need to go about computing the probability mass function which summarizes its randomness. Recalling the connection of $I_{[Z_{x,j}=a|Z_x=m]}$ and $Z_x(t)$ emphasizes that any individual worker would have experienced a sample path of future labor market activity, which could be very short or very long, and so could have experienced a small or large present value of future compensation. We have a choice in studying the probability mass function of (6)—we could either attempt to discover its recursive structure, as we have done with the related random variables, or we could use (6) to generate a large number of realizations via simulation. We will start with the former and discern the necessity of doing the latter. We also have a choice of allocation methods. The theoretical and empirical work that follows utilizes Convention A.

Equations (6) will be seen as new for most forensic economists and actuaries, who have slipped into the habit of thinking about the present value random variable as a number. By taking the mathematical expectations of the left hand sides in (6) and computing or estimating $E\{I_{[Z_{x,j}=a|Z_x=m]}\}$ on the right hand sides, they have been referring to the *expected* present value rather than the present value of the revenue stream. Nowhere has this emphasis been more pronounced than in actuarial science, where the notation for the random variable representing the payment of $1 per year for life is not typically distinguished from its expectation, a_x. We will take up this point again in Section V.

IV. The Probability Mass Function of the Present Value Function

This section, as it describes the probability mass function of the present value function, should also help to fix ideas introduced in the notation of the previous section. Let us assume that *TA*, the truncation age, is 111. The last transition is at age 109.5, and everyone who survives this last transition, governed by either ${}^{a}p_{109}^{a}$ or ${}^{i}p_{109}^{a}$, is dead at 110.5.[1] The probability mass function for any age and initial condition is the function which assigns probabilities to each possible value of the present value random variable. We can do this from first principles. We will work with Convention A and starting active. There are two possibilities: at 109.5, the transition is to active, resulting in payments at 109.5 and 110.5, or the transition is to the inactive or dead states, resulting in no payment. In any case, for the first half of the interval, the beginning state of

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[1] The mathematical computations are insensitive to the specification of this age.

activity is continued (*i.e.*, from age 109 to 109.5). There is a payoff for this half period of .5, which grows and is discounted over this interval, resulting in a present value contribution of $.5\beta$ regardless of the transition. We generically define the probability mass function by the symbols $p_{x,m}(pv)$, where the subscripts are the beginning age and state, *m*, the *pv* arguments are all possible values of the present value, and the value of the function (its range) at each of these *pv* values in the domain is the probability of that value of the present value occurring. Here, the pmf is

$$\begin{gathered} p_{109,a}(.5\beta+.5\beta+.5\beta^3) = p_{109,a}(\beta+.5\beta^3) = {}^a p_{109}^a \\ p_{109,a}(.5\beta) = 1 - {}^a p_{109}^a \,. \end{gathered} \tag{7a}$$

Had we started inactive, there would be only one $.5\beta$ term, and we would either realize a present value of $.5\beta+.5\beta^3$ or nothing. The pmf is

$$\begin{gathered} p_{109,i}(.5\beta+.5\beta^3) = {}^i p_{109}^a \\ p_{109,i}(0) = 1 - {}^i p_{109}^a \,. \end{gathered} \tag{7b}$$

We notice that the domain of $p_{109,a}(\bullet)$ is, for the small *NDR*'s encountered in practice (1% to 3%, say) "essentially" .5 and 1.5, which is exactly the domain encountered when $\beta = 1$ and the present value is counting years of activity. Similarly, the domain of $p_{109,i}(\bullet)$ is, for these small *NDR*'s encountered in practice "essentially" 0 and 1, again exactly the domain encountered when $\beta = 1$ and the present value is counting years of activity. These domains are exactly $0, .5\beta, .5\beta+.5\beta^3, \beta+.5\beta^3$ and do not overlap.[2]

Moving back to age 108, we can begin to see the domains interleave and escalate. Starting active at $x = 108$, there will be $.5\beta$ realized at least. If we go active, we will receive another $.5\beta$, and be active at 109. We can then use the two present values associated with $p_{109,a}(\bullet)$, discounted by β^2 since it is one year in the future, to complete this half of the pmf. Alternatively, we could go inactive at 108, contributing $.5\beta$ and 0 at 108.5, and leaving us with β^2 times the possibilities entailed in $p_{109,i}(\bullet)$. Doing the accounting, showing how each of the two potential contributions for each transition contribute, and then grouping, gives us:

$$\begin{gathered} p_{108,a}(.5\beta+.5\beta+\beta^2\{\beta+.5\beta^3\}) = p_{108,a}(\beta+\beta^3+.5\beta^5) = {}^a p_{108}^a \, {}^a p_{109}^a \\ p_{108,a}(.5\beta+.5\beta+.5\beta^3) = p_{108,a}(\beta+.5\beta^3) = {}^a p_{108}^a ({}^a p_{109}^d + {}^a p_{109}^i) \\ p_{108,a}(.5\beta+\beta^2\{.5\beta+.5\beta^3\}) = p_{108,a}(.5\beta+.5\beta^3+.5\beta^5) = {}^a p_{108}^i \, {}^i p_{109}^a \\ p_{108,a}(.5\beta) = {}^a p_{108}^d + {}^a p_{108}^i ({}^i p_{109}^d + {}^i p_{109}^i) \,. \end{gathered} \tag{8a}$$

The number of distinct points in the domain has doubled; let us define it as $D_{108,a} = \{\beta+\beta^3+.5\beta^5, \beta+.5\beta^3, .5\beta+.5\beta^3+.5\beta^5, .5\beta\}$. The 4 distinct points are

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[2] At age 110, $p_{110,a}(.5\beta) = 1$ and $p_{110,i}(0) = 1$, the domain being "essentially" .5 if active and zero if inactive as we get when counting years of activity.

essentially 2.5, 1.5 and .5, with there being now two distinct but slightly different ways to realize about 1.5. This problem will become worse when we get to 107, and will increase exponentially.

For inactives at age 108 we have

$$
\begin{gathered}
p_{108,i}(0+.5\beta+\beta^2\{\beta+.5\beta^3\}) = p_{108,i}(.5\beta+\beta^3+.5\beta^5) = {}^i p_{108}^a \, {}^a p_{109}^a \\
p_{108,i}(.5\beta+.5\beta^3) = {}^i p_{108}^a ({}^a p_{109}^d + {}^a p_{109}^i) \\
p_{108,i}(0+\beta^2\{.5\beta+.5\beta^3\}) = p_{108,a}(.5\beta^3+.5\beta^5) = {}^i p_{108}^i \, {}^i p_{109}^a \\
p_{108,i}(0) = {}^i p_{108}^d + {}^i p_{108}^i ({}^i p_{109}^d + {}^i p_{109}^i)\,.
\end{gathered}
\tag{8b}
$$

The number of distinct points in this domain has doubled, which we again define as $D_{108,i} = \{.5\beta+\beta^3+.5\beta^5, .5\beta+.5\beta^3, .5\beta^3+.5\beta^5, 0\}$. The domain is essentially 2, 1 and 0, and there are now two distinct but slightly different ways to realize about 1. Again, this problem will become worse when we get to 107, and will increase exponentially thereafter. In fact, Figures 1a, 1b, 2a, and 2b show the general recursion, from which the age $x-1=107$ domain elements and their corresponding probabilities evolve from their age $x=108$ counterparts.

Figure 1a

Domain Elements Probabilities

$$
D_{107,a} = \left\{ \begin{array}{ll}
.5\beta+.5\beta+\beta^2 D_{108,a} & \left\{ \begin{array}{l}
{}^a p_{107}^a \text{ times first } \mathrm{D}_{108,a} \text{ probability} \\
{}^a p_{107}^a \text{ times second } \mathrm{D}_{108,a} \text{ probability} \\
{}^a p_{107}^a \text{ times third } \mathrm{D}_{108,a} \text{ probability} \\
{}^a p_{107}^a \text{ times fourth } \mathrm{D}_{108,a} \text{ probability}
\end{array} \right. \\
.5\beta+0+\beta^2 D_{108,i} & \left\{ \begin{array}{l}
{}^a p_{107}^i \text{ times first } \mathrm{D}_{108,i} \text{ probability} \\
{}^a p_{107}^i \text{ times second } \mathrm{D}_{108,i} \text{ probability} \\
{}^a p_{107}^i \text{ times third } \mathrm{D}_{108,i} \text{ probability} \\
{}^a p_{107}^d + {}^a p_{107}^i \text{ times fourth } \mathrm{D}_{108,i} \text{ probability}
\end{array} \right.
\end{array} \right.
$$

Figure 1b

Domain of $\mathrm{D}_{107,a}$ of $\rho_{107,a}(\bullet)$ (Present Values)	Paths	Essential Values	Range of $\rho_{107,a}(\bullet)$ (Probabilities)	Number of Paths
$.5\beta+.5\beta+\beta^2\{\beta+\beta^3+.5\beta^5\}$	$a\to a\to a\to a$	3.5	${}^a p_{107}^a \, {}^a p_{108}^a \, {}^a p_{109}^a$	1
$.5\beta+.5\beta+\beta^2\{\beta+.5\beta^3\}$	$a\to a\to a\to d;i$	2.5	${}^a p_{107}^a \, {}^a p_{108}^a ({}^a p_{109}^i + {}^a p_{109}^d)$	2
$.5\beta+.5\beta+\beta^2\{.5\beta+.5\beta^3+.5\beta^5\}$	$a\to a\to i\to a$	2.5	${}^a p_{107}^a \, {}^a p_{108}^i \, {}^i p_{109}^a$	1
$.5\beta+.5\beta+\beta^2\{.5\beta\}$	$a\to a\to d;i\to d;i$	1.5	${}^a p_{107}^a \{ {}^a p_{108}^d + {}^a p_{108}^i ({}^i p_{109}^i + {}^i p_{109}^d)\}$	3
$.5\beta+0\beta+\beta^2\{.5\beta+\beta^3+.5\beta^5\}$	$a\to i\to a\to a$	2.5	${}^a p_{107}^i \, {}^i p_{108}^a \, {}^a p_{109}^a$	1
$.5\beta+0\beta+\beta^2\{.5\beta+.5\beta^3\}$	$a\to i\to a\to d;i$	1.5	${}^a p_{107}^i \, {}^i p_{108}^a ({}^a p_{109}^i + {}^a p_{109}^d)$	2
$.5\beta+0\beta+\beta^2\{.5\beta^3+.5\beta^5\}$	$a\to i\to i\to a$	1.5	${}^a p_{107}^i \, {}^i p_{108}^i \, {}^i p_{109}^a$	1
$.5\beta+0\beta+\beta^2\{0\}$	$a\to d;i\to d;i\to d;i$	.5	${}^a p_{107}^d + {}^a p_{107}^i \{ {}^i p_{108}^d + {}^i p_{108}^i ({}^i p_{109}^d + {}^i p_{109}^i)\}$	4

Figure 2a

Domain Elements Probabilities

$$D_{107,i}=\begin{cases} 0+.5\beta+\beta^{2}D_{108,a} & \begin{cases} {}^{i}p^{a}_{107} \text{ times first } \mathrm{D}_{108,a} \text{ probability}\\ {}^{i}p^{a}_{107} \text{ times second } \mathrm{D}_{108,a} \text{ probability}\\ {}^{i}p^{a}_{107} \text{ times third } \mathrm{D}_{108,a} \text{ probability}\\ {}^{i}p^{a}_{107} \text{ times fourth } \mathrm{D}_{108,a} \text{ probability} \end{cases}\\ 0+0+\beta^{2}D_{108,i} & \begin{cases} {}^{i}p^{i}_{107} \text{ times first } \mathrm{D}_{108,i} \text{ probability}\\ {}^{i}p^{i}_{107} \text{ times second } \mathrm{D}_{108,i} \text{ probability}\\ {}^{i}p^{i}_{107} \text{ times third } \mathrm{D}_{108,i} \text{ probability}\\ {}^{i}p^{d}_{107}+{}^{i}p^{i}_{107} \text{ times fourth } \mathrm{D}_{108,i} \text{ probability} \end{cases} \end{cases}$$

Figure 2b

Domain $D_{107,i}$ of $p_{107,i}(\bullet)$ (Present Values)	Paths	Essential Value	Range of $p_{107,i}(\bullet)$ (Probabilities)	Number of Paths
$0\beta+.5\beta+\beta^{2}\{\beta+\beta^{3}+.5\beta^{5}\}$	$i\rightarrow a\rightarrow a\rightarrow a$	3	${}^{i}p^{a}_{107}\,{}^{a}p^{a}_{108}\,{}^{a}p^{a}_{109}$	1
$0\beta+.5\beta+\beta^{2}\{\beta+.5\beta^{3}\}$	$i\rightarrow a\rightarrow a\rightarrow d;i$	2	${}^{i}p^{a}_{107}\,{}^{a}p^{a}_{108}({}^{a}p^{i}_{109}+{}^{a}p^{d}_{109})$	2
$0\beta+.5\beta+\beta^{2}\{.5\beta+.5\beta^{3}+.5\beta^{5}\}$	$i\rightarrow a\rightarrow i\rightarrow a$	2	${}^{i}p^{a}_{107}\,{}^{a}p^{i}_{108}\,{}^{i}p^{a}_{109}$	1
$0\beta+.5\beta+\beta^{2}\{.5\beta\}$	$i\rightarrow a\rightarrow d;i\rightarrow d;i$	1	${}^{i}p^{a}_{107}\{{}^{a}p^{d}_{108}+{}^{a}p^{i}_{108}({}^{i}p^{i}_{109}+{}^{i}p^{d}_{109})\}$	3
$0\beta+0\beta+\beta^{2}\{.5\beta+\beta^{3}+.5\beta^{5}\}$	$i\rightarrow i\rightarrow a\rightarrow a$	2	${}^{i}p^{i}_{107}\,{}^{i}p^{a}_{108}\,{}^{a}p^{a}_{109}$	1
$0\beta+0\beta+\beta^{2}\{.5\beta+.5\beta^{3}\}$	$i\rightarrow i\rightarrow a\rightarrow d;i$	1	${}^{i}p^{i}_{107}\,{}^{i}p^{a}_{108}({}^{a}p^{i}_{109}+{}^{a}p^{d}_{109})$	2
$0\beta+0\beta+\beta^{2}\{.5\beta^{3}+.5\beta^{5}\}$	$i\rightarrow i\rightarrow i\rightarrow a$	1	${}^{i}p^{i}_{107}\,{}^{i}p^{i}_{108}\,{}^{i}p^{a}_{109}$	1
$0\beta+0\beta+\beta^{2}\{0\}$	$i\rightarrow d;i\rightarrow d;i\rightarrow d;i$	0	${}^{i}p^{d}_{107}+{}^{i}p^{i}_{107}\{{}^{i}p^{d}_{108}+{}^{i}p^{i}_{108}({}^{i}p^{d}_{109}+{}^{i}p^{i}_{109})\}$	4

The general pattern is now clear. Starting at 107, with three transitions left (at 107.5, 108.5, and 109.5), there are

$$(1+1)^{TA-x-1}=(1+1)^{111-107-1}=(1+1)^{3}=\binom{3}{0}1^{3}+\binom{3}{1}1^{2}1^{1}+\binom{3}{2}1^{1}1^{2}+\binom{3}{3}1^{3}=8\text{ points}$$

in each of the domains of $D_{107,a}$ and $D_{107,j}$. Binomial coefficients capture the number of ways in which each of the essential values occurs, there being 3+1=4 such essential values for each domain of 8 points. The domains of $D_{107,a}$ and $D_{107,i}$ are non-overlapping, being essentially on half-integers and integers, respectively, although this distinction will blur as the number of periods less than the terminal age, three above, increases. The link between the domains and ranges one year apart is indicated in Figures 1a and 2a, holds in general, and expresses the recursion of the present value function.

This recursion, although true, provides only insight into the mathematical structure and cannot be exploited for computational purposes, unless $\beta = 1$, in which case the number of distinct points in the domain does not increase exponentially, but equals $(TA - x)$. For a person age *x*, the number of distinct points, $2^{(TA-x-1)}$, becomes a number which is impossible to compute and store: at age 50, $2^{111-50-1} = 2^{60} = 2^{50}2^{10}$, and 2^{50} megabytes of information will never be computable. We state this as the

Present Value Function Properties and Recursion: The present value functions $p_{x,a}(\bullet)$ and $p_{x,i}(\bullet)$ for a person exact age *x* will contain $2^{(TA-x-1)}$ distinct possible values (points in the domains $D_{x,a}$ and $D_{x,i}$, respectively). These arise from $2^{(TA-x)} - 1$ sample paths, giving rise to $(TA - x)$ essential values, which arise from grouping

$$\binom{TA-x-1}{0}, \binom{TA-x-1}{1}, \binom{TA-x-1}{2}, \ldots, \binom{TA-x-1}{TA-x-2}, \binom{TA-x-1}{TA-x-1}$$

of the distinct values together (ignoring the powers of β). While computationally infeasible, the domain of $D_{x,a}$ follows from the domains of $D_{x+1,a}$ and $D_{x+1,i}$ by use of the symbolic relation where the first half of $D_{x,a} = .5\beta + .5\beta + \beta^2 D_{x+1,a}$ and the second half of $D_{x,a} = .5\beta + 0\beta + \beta^2 D_{x+1,i}$ defined above, and the same is true for the first half of $D_{x,i} = 0\beta + .5\beta + \beta^2 D_{x+1,a}$ and the second half of $D_{x,i} = 0\beta + 0\beta + \beta^2 D_{x+1,i}$. The ranges of $p_{x,a}(\bullet)$ (respectively, $p_{x,i}(\bullet)$) follow from the ranges of $p_{x+1,a}(\bullet)$ and $p_{x+1,i}(\bullet)$ by applying to them the transition probabilities ${}^a p_x^a$, ${}^a p_x^i$ and ${}^a p_x^d$ (respectively, ${}^i p_x^a$, ${}^i p_x^i$ and ${}^i p_x^d$) as indicated in Figures 1a and 1b (respectively, Figures 2a and 2b).

In light of the analytic complexity and computational infeasibility of the present value function, it might appear unlikely that there would be a useful and elegant recursion to be found. The next section develops such a recursion for the means of the present value random variables.

V. Present Value Function Mean Recursions

Let

$$(9) \quad {}^a epv_x^a \equiv E(PVA_{x,a}) = \sum_{s \in D_{x,a}} s p_{x,a}(s) \text{ and } {}^i epv_x^a \equiv E(PVA_{x,i}) = \sum_{s \in D_{x,i}} s p_{x,i}(s)$$

be the means or mathematical expectations of the probability mass functions defined above. We now define, in exactly the same way as we did for $PVA_{x,a}$ and $PVA_{x,i}$, the present value functions paying in the active state, the new present value functions $PVI_{x,a}$ and $PVI_{x,i}$ which pay $1 for each year in the inactive state. These are not likely to be nearly as important, but their inclusion actually simplifies the theory which follows, which is enough justification. Additionally, the sum of these present values gives a standard annuity (which pays in both states, *i.e.*, while alive); and annuities are of interest. Finally, the value of an income stream for all years while inactive might have independent interest, (*e.g.* in providing discretionary income or describing income requirements when there is no labor force income).

We will determine the present value and expected present value of years of inactivity

$$(4a)' \quad YI_{x,a} = \sum_{j=.5}^{j=TA-x-.5} I_{[Z_{x,j}=i|Z_x=a]} \text{ and (4b)' } YI_{x,i} = .5 + \sum_{j=.5}^{j=TA-x-.5} I_{[Z_{x,j}=i|Z_x=i]} \, .$$

The present values are defined much like those for years of activity. We use only the Convention A below, and so only give equations corresponding to its timing convention:

$$(6a)' \quad \begin{aligned} PVI_{x,a} &= \sum_{j=.5}^{j=TA-x-1.5} I_{[Z_{x,j}=i|Z_x=a]} \left\{ aei_{j+1}.5\left(\frac{1+g_i}{1+r}\right)^{j+1} + aei_j.5\left(\frac{1+g_i}{1+r}\right)^{j} \right\} \text{ for } x < TA-1 \\ &= \sum_{j=.5}^{j=TA-x-1.5} I_{[Z_{x,j}=i|Z_x=a]} \left\{ aei_{j+1}.5\left(\beta_i^2\right)^{j+1} + aei_j.5\left(\beta_i^2\right)^{j} \right\} \\ PVI_{x,a} &= 0 \text{ for } x = TA-1 \end{aligned}$$

$$(6b)' \quad \begin{aligned} PVI_{x,i} &= .5\left(\frac{1+g_i}{1+r}\right)^{.5} \sum_{j=.5}^{j=TA-x-1.5} I_{[Z_{x,j}=i|Z_x=i]} \left\{ aei_{j+1}.5\left(\frac{1+g_i}{1+r}\right)^{j+1} + aei_j.5\left(\frac{1+g_i}{1+r}\right)^{j} \right\} \text{ for } x < TA-1 \\ &= .5\beta_i + \sum_{j=.5}^{j=TA-x-1.5} I_{[Z_{x,j}=i|Z_x=i]} \left\{ aei_{j+1}.5\left(\beta_i^2\right)^{j+1} + aei_j.5\left(\beta_i^2\right)^{j} \right\} \\ PVI_{x,1} &= .5\left(\frac{1+g_i}{1+r}\right) = .5\beta_i \text{ for } x = TA-1 \, . \end{aligned}$$

We allow for a different growth rate and a different age earnings profile, with the symbols g_i and $\{aei_j\}$. Expectations will be

$${}^{a}epv_x^i \equiv E(PVI_{x,a}) = \sum_{s \in D_{x,a}} sp_{x,a}(s) \text{ and } {}^{i}epv_x^i \equiv E(PVI_{x,i}) = \sum_{s \in D_{x,i}} sp_{x,i}(s) \, ,$$

where we understand that, if we were to develop these probability mass functions, we should adopt superscripts to distinguish these functions and their domains from those given in (9).

We now record the key equations for the mean value recursions, which will be combined into matrix form below.

(10a) $${}^{a}epv_x^a = .5ae_x\beta + .5\beta ae_x\,{}^{a}p_x^a + \beta^2\{{}^{a}epv_{x+1}^a\,{}^{a}p_x^a + {}^{i}epv_{x+1}^a\,{}^{a}p_x^i\}$$

(10b) $${}^{i}epv_x^a = 0ae_x\beta + .5\beta ae_x\,{}^{i}p_x^a + \beta^2\{{}^{a}epv_{x+1}^a\,{}^{i}p_x^a + {}^{i}epv_{x+1}^a\,{}^{i}p_x^i\}$$

(10c) $${}^{a}epv_x^i = 0aei_x\beta_i + .5\beta_i aei_x\,{}^{a}p_x^i + \beta_i^2\{{}^{i}epv_{x+1}^i\,{}^{a}p_x^i + {}^{a}epv_{x+1}^i\,{}^{a}p_x^a\}$$

(10d) $${}^{i}epv_x^i = .5aei_x\beta_i + .5\beta_i aei_x\,{}^{i}p_x^i + \beta_i^2\{{}^{i}epv_{x+1}^i\,{}^{i}p_x^i + {}^{a}epv_{x+1}^i\,{}^{i}p_x^a\}$$

We consider (10a). The present value of all future activity, starting active, will have several components, the first of which is a payment ½ of a year from age *x*, hence the ½ year discount factor β on the \$1 for ½ of a period, and therefore the .5, augmented by the age earnings factor, ae_x (which would typically be normalized to 1 for age *x*). Next, if there is a transition resulting in remaining active, which occurs at *x* + ½, then by our Convention A, .5 of this is paid then (this .5 must again be discounted back to *x* by β, and it occurs with probability ${}^{a}p_x^a$, so that its expected value is $.5\beta ae_x\,{}^{a}p_x^a$, while .5 is paid 1 year later, and so will be embedded in the present value ${}^{a}epv_{x+1}^a$. The remaining payments will occur at *x*+1.5 and later, and their present value, as of *x*+1, ${}^{a}epv_{x+1}^a$ if the transition at *x* + ½ is to active, and is ${}^{i}epv_{x+1}^a$ if the *x* + ½ transition is to inactive. These happen with probabilities ${}^{a}p_x^a$ and ${}^{a}p_x^i$, so that the expected present value at *x*+1 is $\{{}^{a}epv_{x+1}^a\,{}^{a}p_x^a + {}^{i}epv_{x+1}^a\,{}^{a}p_x^i\}$. However, we need to adjust these *x*+1 values back to age *x*, which requires discounting for two half periods, hence by β^2.

The derivation of (10b)-(10d) are similar, where in (10c)-(10d) we are counting inactivity rather than activity. Recursions (10a)-(10d) may be gathered into matrix equalities, where their structure becomes clearer. Gathering the four equations gives

(11a) $$\begin{pmatrix} {}^{a}epv_x^a & {}^{i}epv_x^a \\ {}^{a}epv_x^i & {}^{i}epv_x^i \end{pmatrix} = \begin{pmatrix} \beta & 0 \\ 0 & \beta_i \end{pmatrix} \begin{pmatrix} ae_x & 0 \\ 0 & aei_x \end{pmatrix} \left\{ .5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + .5\begin{pmatrix} {}^{a}p_x^a & {}^{i}p_x^a \\ {}^{a}p_x^i & {}^{i}p_x^i \end{pmatrix} \right\} + \begin{pmatrix} \beta & 0 \\ 0 & \beta_i \end{pmatrix}^2 \begin{pmatrix} {}^{a}epv_{x+1}^a & {}^{i}epv_{x+1}^a \\ {}^{a}epv_{x+1}^i & {}^{i}epv_{x+1}^i \end{pmatrix} \begin{pmatrix} {}^{a}p_x^a & {}^{i}p_x^a \\ {}^{a}p_x^i & {}^{i}p_x^i \end{pmatrix}.$$

Giving each matrix its self-evident definition in (11a), and using the symbol * to denote matrix multiplication (since the juxtaposition of BAE_x could be confused with $B * AE_x$), results in

(11b) $$EPV_x = B * AE_x * .5\{I + P_x\} + B^2 * EPV_{x+1} * P_x,$$

which we record as the *Expected Present Value Theorem.* When labor force participation follows the Markov model, the expected present value of each base (age *x*) year's \$1 of wages and fringe benefits is given by the (1,1) element of $EPV_x = B * AE_x * .5\{I + P_x\} + B^2 * EPV_{x+1} * P_x$ when starting active, and by the (1,2) element when starting inactive. The expected present value of each base (age *x*) year's \$1 in the inactive state is given by the (2,1) element when starting active, and by the (2,2) element when starting inactive.

Several comments are in order.

Comment 1. While derived by a forward looking argument, the great computational use of the result runs time (age) in the opposite direction. We need some value for EPV_{x+1} at some age x+1 on the right hand side to be able to use the equation to simply and quickly calculate all previous values $EPV_x, EPV_{x-1}, \cdots$. The natural age would equate x+1 to $TA-1$, since at that age, only ½ year of life remains. When measuring activity, one receives payment for $.5\,ae_{TA-1}$ years discounted by β , if active at $TA-1$ and 0 otherwise with certainty. The same is true when measuring inactivity—5$\,aei_{TA-1}$ is received and discounted at β_i , if inactive at $TA-1$ and nothing otherwise. Thus, to start the recursion we have the boundary condition

$$EPV_{TA-1} = \begin{pmatrix} .5\beta ae_{TA-1} & 0 \\ 0 & .5\beta_i aei_{TA-1} \end{pmatrix}. \tag{12}$$

Comment 2. If $TA-1$ is taken to be 110, because of the values of AE_x and P_x between x = 80 and x = 109, the values in EPV_{80} are quite insensitive to either the choice of $TA-1$ or the values chosen for EPV_{TA-1}.

Comment 3. Our present value or Scogden Tables, like the Ogden Tables, are calculated with no age earnings profile, *i.e.* with $AE_x = I$ for all ages x. Nevertheless, (11b) shows that it is quite straightforward to introduce age earnings sequences $\{ae_{x;j}\}_{j=0}^{j=TA-x-1} \equiv \{ae_{x+j}\}_{j=0}^{j=TA-x-1}$ and $\{aei_{x;j}\}_{j=0}^{j=TA-x-1} \equiv \{aei_{x+j}\}_{j=0}^{j=TA-x-1}$ normalized with $ae_x = aei_x = 1$ into the analysis.

Comment 4. When $\beta = \beta_i = 1$ so that $B = I$, and $AE_x = I$, the recursion reduces to $EPV_x = .5\{I + P_x\} + EPV_{x+1}P_x$. But this is the same recursion which E_x obeys: $E_x = .5\{I + P_x\} + E_{x+1}P_x$, (*c.f.* Skoog, 2002; Foster and Skoog. 2004). Hence with no discounting, since

$$EPV_{TA-1} = \begin{pmatrix} .5 & 0 \\ 0 & .5 \end{pmatrix} = E_{TA-1},$$

it follows that $E_x = EPV_x$, where

$$E_x \equiv \begin{pmatrix} {}^a e_x^a & {}^i e_x^a \\ {}^a e_x^i & {}^i e_x^i \end{pmatrix}$$

is the matrix of expected time in the upper right superscripted state, given that one has started at age x in the upper left superscripted state. Consequently, we have proved that in this case $E_x = EPV_x$.

Comment 5. Reflecting the dependence of the expected present values ${}^m epv_x^n$ on the growth rates and the interest rate, for any initial and final states m and n,

$EPV_x(g,g_i,r)$ is continuous (this is intuitive from the recursion, and is demonstrated below). Consequently the matrix function $EPV_x(g,g_i,r)$ is continuous as g and g_i approach r, and the limit is well known:

$$\lim_{\substack{g\to r \\ g_i\to r}} EPV_x(g,g_i,r) = EPV_x(r,r,r) = E_x . \tag{13}$$

Aside from the elegant theoretical interpretation of EPV_x as an extension of a well known function off its boundary, the equality is useful in checking a computational algorithm for $EPV_x(g,g_i,r)$.

Comment 6. An exact expression for the means provides computational checks when the entire probability mass function is generated by simulation, as it must be when its general properties, aside from its mean, are being studied. In this case, the question will arise as to whether a simulated sample size is large enough to be assured that the strong law of large numbers has taken effect. By calculating the EPV_x for the simulation and comparing the generated result with known answer *via* the recursion, one can check for the appropriateness of sample size—if the simulated mean is not accurate, a larger sample size is needed. Conversely, there is no need to run the computer for a week if the means converge with a sample size in the tens of thousands, which would be computed in seconds for one exact age.

Comment 7. Since diagonal matrices B and AE_x commute, the key recursion

$$EPV_x = B * AE_x * .5\{I + P_x\} + B^2 * EPV_{x+1} * P_x$$

may be re-written as

$$EPV_x = AE_x B\{.5I + .5P_x\} + B^2 EPV_{x+1} P_x \tag{14a}$$

(not requiring * to resolve ambiguity).

Comment 8. Setting $AE_x = I$ results in the equation:

$$EPV_x = B\{.5I + .5P_x\} + B^2 EPV_{x+1} P_x \tag{14b}$$

Comment 9. The information in recursion (14a) may be re-expressed in a form that makes the calculations more familiar to forensic economists, by repeated advancement of the age and substitution. Re-writing (14a) for ages x, x+1 and x+2, and noting that this may be continued to $x + (TA - x - 1)$, we have:

$$\begin{aligned} EPV_x &= AE_x B\{.5I + .5P_x\} + B^2 EPV_{x+1} P_x \\ EPV_{x+1} &= AE_{x+1} B\{.5I + .5P_{x+1}\} + B^2 EPV_{x+2} P_{x+1} \\ EPV_{x+2} &= AE_{x+2} B\{.5I + .5P_{x+2}\} + B^2 EPV_{x+3} P_{x+2} . \end{aligned}$$

Substituting the second equation into the first results in

$$EPV_x = AE_x B\{.5I + .5P_x\} + B^2[AE_{x+1} B\{.5I + .5P_{x+1}\} + B^2 EPV_{x+2} P_{x+1}]P_x ,$$

while substituting the third expression yields

$$\begin{aligned} &EPV_x = AE_x B\{.5I + .5P_x\} + \\ &B^2 \left\langle AE_{x+1} B\{.5I + .5P_{x+1}\} + B^2[AE_{x+2} B(.5I + .5P_{x+2}) + B^2 EPV_{x+3} P_{x+2}]P_{x+1} \right\rangle P_x \end{aligned} \tag{15}$$

Before expanding (15), we define, as in Skoog (2002), the ${}_j\Pi_x$ matrices as:

$${}_0\Pi_x = I, {}_1\Pi_x = P_x, {}_2\Pi_x = P_{x+1}P_x, \cdots, {}_j\Pi_x = P_{x+j-1} \cdots P_{x+1}P_x \text{ for any } j. \tag{16}$$

Elements of the matrices

$${}_j\Pi_x \equiv \begin{pmatrix} {}^a_j\pi^a_x & {}^i_j\pi^a_x \\ {}^a_j\pi^i_x & {}^i_j\pi^i_x \end{pmatrix}$$

are probabilities that a worker in the state specified by the upper left superscripted state at age x will be in the state specified by the upper right superscript at age $x + j$. This is true by definition for ${}_1\Pi_x$, and is evident if the terms in ${}_2\Pi_x = P_{x+1}P_x$ on the right hand side are written out.

We return to (15), which needs expansion. Two groupings are possible, each with an interesting economic interpretation. The first expansion organizing the flows in age intervals (x, x+1], (x +1, x+2], (x +2, x+3], … is

$$\begin{aligned} EPV_x &= \{.5B * AE_x + .5B * AE_x P_x\} + \{.5B^3 AE_{x+1} P_x + .5B^3 AE_{x+1} P_{x+1} P_x\} + \\ &\{.5B^5 AE_{x+2} P_{x+1} P_x + .5B^5 AE_{x+2} P_{x+2} P_{x+1} P_x\} + B^6 EPV_{x+3} P_{x+2} P_{x+1} P_x \\ &= B * AE_x \{.5I + .5P_x\} + B^3 AE_{x+1} \{.5P_x + .5P_{x+1} P_x\} + \\ &B^5 AE_{x+2} \{.5P_{x+1} P_x + .5P_{x+2} P_{x+1} P_x\} + B^6 EPV_{x+3} P_{x+2} P_{x+1} P_x \\ &= B * AE_x \{.5\,{}_0\Pi_x + .5\,{}_1\Pi_x\} + B^3 AE_{x+1} \{.5\,{}_1\Pi_x + .5\,{}_2\Pi_x\} + \\ &B^5 AE_{x+2} \{.5\,{}_2\Pi_x + .5\,{}_3\Pi_x\} + B^6 EPV_{x+3}\,{}_3\Pi_x . \end{aligned} \tag{17a}$$

Thus, continuing the substitutions in (17a) we have

$$EPV_x = \sum_{j=1}^{j=TA-x} B^{2j-1} * AE_{x+j-1} \{.5\,{}_{j-1}\Pi_x + .5\,{}_j\Pi_x\} \tag{17b}$$

where the second payment and the first payment of .5 from two adjacent transitions are combined.

The second expansion, organizing the flows in transition intervals (x, x+.5], (x+.5, x+1.5], (x+1.5, x+2.5],…, resulting from re-arranging the grouping of terms in (15), is

$$
\begin{aligned}
EPV_x &= .5B * AE_x + \{.5B * AE_x P_x + .5B^3 AE_{x+1} P_x\} + \\
&\quad \{.5B^3 AE_{x+1} P_{x+1} P_x + .5B^5 AE_{x+2} P_{x+1} P_x\} + \\
&\quad \{.5B^5 AE_{x+2} P_{x+2} P_{x+1} P_x + B^6 EPV_{x+3} P_{x+2} P_{x+1} P_x\} \\
&= .5B * AE_x + \{.5B * AE_x + .5B^3 AE_{x+1}\}\, {}_1\Pi_x + \\
&\quad \{.5B^3 AE_{x+1} + .5B^5 AE_{x+2}\}\, {}_2\Pi_x + \\
&\quad \{.5B^5 AE_{x+2} + B^6 EPV_{x+3}\}\, {}_3\Pi_x \\
&= \ .5B * AE_x + \sum_{j=1}^{j=TA-x} \{.5B^{2j-1} * AE_{x+j-1} + .5B^{2j+1} AE_{x+j}\}\, {}_j\Pi_x .
\end{aligned}
\tag{18}
$$

The first term in (18) is the second payment made at $x + .5$ from the previous, $x - .5$ transition. The terms in the summation capture the two payments of .5 generated from the x+.5, x+1.5,… transitions. In both representations, the half period discount factors $B, B^3, \cdots$ are centered on mid-points, and since multiplication and addition over a finite number of terms preserve continuity of the arguments, this establishes the continuity of EPV_x in β, β_i claimed in equation (13).

Comment 10. We provide tables below which may be used with interpolation between ages for the injured or decedent when damages are computed as of the accident date. Support for calculation as of this date is found in the famous footnote 22 of *Jones and Laughlin Steel Corp. v. Pfeifer*, 462 U.S. 523. In such a case, under federal law, the amount so calculated would be advanced by a pre-judgment interest factor. In most state cases, however, the damage period is split between pre-trial losses, on which no discounting or augmenting (pre-judgment interest) is calculated, and future losses, which are discounted to present value. In such a case, where the damage flows are treated asymmetrically, either an approximation would be needed or an extension of these tables would need to be computed. For example, assuming that no more than 7 years lapse between the tort and the trial, for a personal injury case where the plaintiff is known to have survived to the trial date, the probabilities of being active and inactive in each of the pre-trial years could be computed in a separate or extended chart, zeroing out mortality. Similar calculations incorporating mortality would be pertinent in state wrongful death cases, where pre-trial survival would not be known. Such probabilities might be useful in refining calculations of pre-trial loss, and in determining the weights for a $PVA_{x,a}$ - $PVA_{x,i}$ weighted average post-trial calculation. In any event, the tables provided below, with or without such adjustments, will provide both a reality check on present value calculations as well as a clear statement of the underlying variation about the expected present value which forensic economists currently report.

Comment 11. Since the present value calculation is nonlinear in the *NDR*, the following tables permit a quick approximate answer to the question about how present value calculations would change with a small increase or decrease in the *NDR*.

VI. Tables[3]

Table 1 shows some characteristics of $PVA_{x,a}$ for $x = 16,...,75$ under Convention A using (6a) for initially active male high school graduates (assuming age-earnings factors ae_j of 1). This table contains simulation estimates[4] for the expected present value, median present value, standard deviation, skewness, kurtosis and the 10th, 25th, 75th, and 90th percentile points of the present value pmf at *NDR*'s of 0, .0050, .0100, .0125, .0150, .0175, .0200, .0250, .0300, .0350, .0400. Table 1 is based on simulating 30,000 sample paths at each age *x*. Certain obvious conclusions, present empirically in Table 1, follow from the foregoing mathematical treatment and discussion: (1) Expected present value varies inversely with *NDR*; (2) Expected present value varies inversely with age; and (3) As with life annuities and annuities certain, decrements (measured in absolute value) in expected present values become smaller for equal increases in the *NDR*.

Beyond the magnitudes of the entries themselves, Table 1 reveals four less obvious insights about present value pmf's. (1) Given age, the standard deviation of the present value random variable declines with the *NDR*. Figures 3a–3c show present value pmf's at age 30 for *NDR* = 0, .02, and .04 -- declining standard deviations with higher *NDR*'s are apparent in these figures which have the same scaling of axes in order to facilitate comparability. In addition, at young ages the standard deviation is almost three times larger when *NDR* = 0 than it is when *NDR* = .04. At age 40, the standard deviation is twice as large when computed at *NDR* = 0 as at *NDR* =.04; it is approximately 1.4 times larger at age 60. (2) Given age, skewness declines (algebraically) with higher *NDR*, *i.e.*, if the skewness coefficient is negative at a low *NDR*, it will be smaller (bigger in absolute value) at larger *NDR*'s; and, if skewness is positive at a low *NDR*, it will be smaller at larger *NDR*'s and may become negative. Figures 3a-3c show this relationship—pmf's become more skewed to the left as

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[3] Table 1, for initially active male high school graduates, is quite long. A comprehensive set of similar tables for various combinations of gender, initial labor force status, and several educational levels could be produced and would be more useful for forensic work. However, the basic contribution of this paper is theoretical; and it provides the foundation for the first glimpse of the characteristics and appearance of the probability mass functions of the present value random variable. We note that Table 1 is immediately useful in forensic work for initially active male high school graduates.

[4] We use transition probabilities from the Current Population Survey as extracted by Kurt Krueger for the period 1998-2008 until age 80 and gradually let active-to-active and inactive-to-active transition probabilities decline to zero as age approaches *TA*. These transition probabilities for high school graduates were first used in a paper by Skoog, Krueger, and Ciecka (2009) and will be part of a new comprehensive set of extended worklife tables by Skoog, Ciecka and Krueger. These new tables will contain worklives by age, gender, labor force status, and education as well as the full range of activity-related characteristics (i.e., median, mode, standard deviation, skewness, kurtosis, and various percentile points). For mortality, we use *National Vital Statistics Reports* (2007) which contains mortality probabilities as of 2004.

NDR increases. Present value pmf's display negative skewness at younger ages and positive skewness at older ages with means typically less than medians at young ages and the reverse at older ages. These relationships are reminiscent of survival data where life expectancy is less than median additional years of life for young people, but life expectancy exceeds median years of life at older ages. (3) Kurtosis increases with *NDR* until approximately age 48 and varies inversely with *NDR* thereafter. We note that approximate normality occurs when skewness is close to zero and kurtosis close to 3.00. (4) Percentile points also may be of interest. For example, at age 30 and using the 25th and 75th percentile entries in Table 1, the probability is 50% that the present value random variable will lie within the interval (24.50, 34.50) with the expected present value being 29.30 when *NDR* = 0. This 50% probability interval tightens to (18.79, 24.63) with the expected value being 21.29 when *NDR* = .02. The probability interval is (14.77, 18.56) when *NDR* = .04, and expected present value is 16.26. We note that the expected present values themselves lie closer to the left end points of their respective intervals as the *NDR* increases, thereby reflecting the relationship between skewness and *NDR*.

The final two columns of Table 1 contain bootstrap estimates of the means of present value and corresponding standard deviations for the sample means based on *Current Population Survey* sample sizes, 100 bootstrap replications, and 3,000 simulations at each age and each *NDR*. We note that each row in Table 1 contains two standard deviations: the first measures variability in the present value random variable itself and the second measures variability in the sample mean of present value. As an example, consider age 30 and *NDR* = .02. Table 1 shows an expected present value of 21.29 and a standard deviation of the present value random variable of 4.92. This standard deviation measures the intrinsic variability in present value and includes the impacts of events like premature death and early departure from the labor force as well as being long lived and staying in the labor force to old age. The bootstrap estimate of the expected present value is 21.27, a value quite close to 21.29, the expected present value. The bootstrap estimate of the standard error of the sample mean present value is only .13. The latter figure, being small, tells us that the distribution of the sample expected present value is very compact. If, for example, a forensic economist were to provide information about the average present value and its accuracy, then the estimate of the average would be subject to only a small amount of error. However, one must remember that the standard deviation of the present value random variable itself is much larger, reflecting the substantial variability around the expected present value as captured by the distributional characteristics in Table 1.

Table 2 enables us to compare EPV_x from simulations with the exact result from the *Expected Present Value Theorem*. Table 2 shows errors in simulated means at each age relative to the exact EPV_x. In most cases, errors are less than three-tenths of one percent; thus providing strong support for the accuracy of the simulations and bootstrap estimates in Table 1.

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
16	0.0000	39.91	41.50	9.49	-1.02	5.04	28.50	35.50	46.50	50.50	39.85	0.32
16	0.0050	35.40	36.59	7.99	-1.16	5.47	25.70	31.48	40.63	43.99	35.34	0.28
16	0.0100	31.58	32.66	6.79	-1.28	5.91	23.42	28.33	36.02	38.78	31.53	0.25
16	0.0125	29.89	30.91	6.28	-1.33	6.13	22.38	26.88	34.04	36.51	29.84	0.24
16	0.0150	28.33	29.30	5.83	-1.38	6.33	21.35	25.59	32.18	34.50	28.28	0.23
16	0.0175	26.88	27.79	5.43	-1.43	6.53	20.44	24.34	30.51	32.58	26.84	0.22
16	0.0200	25.55	26.42	5.06	-1.47	6.70	19.53	23.20	28.94	30.81	25.51	0.21
16	0.0250	23.16	23.94	4.44	-1.53	7.00	17.89	21.12	26.15	27.79	23.12	0.20
16	0.0300	21.10	21.80	3.94	-1.57	7.21	16.44	19.29	23.77	25.22	21.06	0.19
16	0.0350	19.31	19.94	3.53	-1.58	7.32	15.14	17.68	21.72	22.99	19.28	0.18
16	0.0400	17.75	18.32	3.19	-1.58	7.33	13.95	16.25	19.94	21.09	17.72	0.18
17	0.0000	39.28	40.50	9.40	-1.01	4.94	27.50	34.50	45.50	49.50	39.21	0.20
17	0.0050	34.94	36.12	7.94	-1.14	5.35	25.27	31.12	40.22	43.49	34.88	0.17
17	0.0100	31.25	32.32	6.77	-1.26	5.77	23.01	28.09	35.78	38.46	31.20	0.15
17	0.0125	29.62	30.62	6.27	-1.32	5.98	21.99	26.72	33.81	36.24	29.57	0.14
17	0.0150	28.10	29.08	5.83	-1.37	6.18	21.05	25.43	32.02	34.21	28.06	0.13
17	0.0175	26.70	27.63	5.43	-1.41	6.36	20.13	24.25	30.39	32.36	26.66	0.13
17	0.0200	25.41	26.30	5.07	-1.45	6.54	19.29	23.12	28.83	30.72	25.37	0.12
17	0.0250	23.08	23.88	4.46	-1.52	6.84	17.74	21.07	26.11	27.66	23.05	0.11
17	0.0300	21.07	21.79	3.96	-1.56	7.06	16.35	19.28	23.78	25.18	21.04	0.10
17	0.0350	19.32	19.99	3.55	-1.59	7.18	15.08	17.69	21.76	22.99	19.29	0.09
17	0.0400	17.79	18.40	3.21	-1.59	7.23	13.93	16.31	20.01	21.13	17.77	0.09
18	0.0000	38.57	39.50	9.35	-0.99	4.94	27.50	33.50	44.50	48.50	38.58	0.22
18	0.0050	34.41	35.58	7.92	-1.14	5.36	24.74	30.58	39.67	43.02	34.42	0.19
18	0.0100	30.87	31.94	6.78	-1.27	5.80	22.69	27.66	35.31	38.11	30.87	0.16
18	0.0125	29.29	30.31	6.29	-1.32	6.02	21.74	26.34	33.50	35.92	29.29	0.15
18	0.0150	27.83	28.78	5.85	-1.38	6.23	20.84	25.12	31.73	34.04	27.83	0.14
18	0.0175	26.48	27.41	5.46	-1.43	6.43	19.96	23.98	30.09	32.23	26.48	0.13
18	0.0200	25.23	26.09	5.10	-1.48	6.63	19.15	22.90	28.66	30.54	25.22	0.12
18	0.0250	22.97	23.79	4.50	-1.56	6.97	17.66	20.95	26.01	27.64	22.96	0.11
18	0.0300	21.01	21.76	4.00	-1.61	7.24	16.30	19.25	23.74	25.08	21.00	0.10
18	0.0350	19.31	20.00	3.59	-1.65	7.43	15.06	17.72	21.78	22.98	19.29	0.09
18	0.0400	17.81	18.45	3.25	-1.68	7.53	13.95	16.37	20.07	21.10	17.80	0.08
19	0.0000	37.94	39.50	9.24	-0.98	4.84	26.50	33.50	44.50	48.50	37.94	0.24
19	0.0050	33.94	35.03	7.85	-1.12	5.26	24.33	30.00	39.10	42.29	33.94	0.20
19	0.0100	30.52	31.58	6.73	-1.24	5.69	22.34	27.31	35.04	37.74	30.52	0.17
19	0.0125	29.00	29.98	6.26	-1.30	5.90	21.45	26.07	33.21	35.63	29.00	0.16
19	0.0150	27.59	28.55	5.83	-1.36	6.12	20.53	24.88	31.49	33.68	27.58	0.15
19	0.0175	26.28	27.17	5.44	-1.41	6.32	19.73	23.76	29.99	31.91	26.27	0.14
19	0.0200	25.06	25.96	5.09	-1.46	6.52	18.93	22.73	28.53	30.39	25.05	0.13
19	0.0250	22.86	23.66	4.49	-1.54	6.88	17.48	20.86	25.96	27.48	22.86	0.11
19	0.0300	20.96	21.70	4.00	-1.61	7.18	16.16	19.19	23.71	25.01	20.95	0.10
19	0.0350	19.29	19.99	3.59	-1.66	7.41	14.98	17.69	21.81	22.93	19.28	0.09
19	0.0400	17.83	18.49	3.25	-1.70	7.56	13.91	16.38	20.09	21.07	17.82	0.08
20	0.0000	37.39	38.50	9.19	-0.96	4.79	25.50	32.50	43.50	47.50	37.32	0.21
20	0.0050	33.53	34.72	7.83	-1.10	5.20	23.89	29.76	38.80	42.06	33.47	0.18
20	0.0100	30.23	31.30	6.73	-1.24	5.64	22.04	27.04	34.66	37.27	30.18	0.15
20	0.0125	28.76	29.77	6.26	-1.30	5.86	21.12	25.84	33.00	35.32	28.71	0.14
20	0.0150	27.39	28.36	5.83	-1.36	6.09	20.36	24.68	31.35	33.52	27.35	0.13

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
20	0.0175	26.12	27.04	5.45	-1.42	6.31	19.55	23.64	29.79	31.80	26.08	0.12
20	0.0200	24.93	25.87	5.10	-1.47	6.52	18.79	22.62	28.33	30.19	24.90	0.12
20	0.0250	22.80	23.64	4.50	-1.57	6.93	17.40	20.82	25.88	27.33	22.77	0.10
20	0.0300	20.94	21.73	4.00	-1.65	7.29	16.14	19.19	23.66	24.97	20.91	0.09
20	0.0350	19.30	20.06	3.59	-1.72	7.58	14.98	17.78	21.80	22.87	19.28	0.08
20	0.0400	17.87	18.58	3.25	-1.77	7.81	13.97	16.48	20.11	21.04	17.85	0.08
21	0.0000	36.60	37.50	9.04	-0.93	4.67	25.50	31.50	42.50	46.50	36.60	0.23
21	0.0050	32.92	34.05	7.72	-1.07	5.07	23.34	29.05	38.14	41.37	32.91	0.20
21	0.0100	29.75	30.79	6.65	-1.20	5.50	21.56	26.51	34.25	36.89	29.75	0.17
21	0.0125	28.34	29.34	6.19	-1.26	5.72	20.74	25.38	32.45	34.86	28.33	0.16
21	0.0150	27.02	28.03	5.78	-1.33	5.94	19.93	24.28	30.96	33.05	27.02	0.15
21	0.0175	25.79	26.74	5.40	-1.38	6.17	19.24	23.26	29.53	31.45	25.79	0.14
21	0.0200	24.65	25.53	5.06	-1.44	6.39	18.49	22.34	28.13	29.92	24.65	0.13
21	0.0250	22.59	23.45	4.47	-1.54	6.82	17.19	20.55	25.60	27.13	22.59	0.11
21	0.0300	20.78	21.61	3.98	-1.63	7.21	15.94	19.03	23.53	24.75	20.78	0.10
21	0.0350	19.20	19.94	3.57	-1.71	7.56	14.86	17.64	21.63	22.77	19.20	0.09
21	0.0400	17.80	18.51	3.23	-1.77	7.85	13.87	16.42	20.06	20.97	17.80	0.08
22	0.0000	35.85	37.50	8.91	-0.89	4.54	24.50	31.50	41.50	45.50	35.89	0.21
22	0.0050	32.32	33.37	7.63	-1.03	4.90	22.81	28.48	37.46	40.69	32.36	0.18
22	0.0100	29.28	30.37	6.58	-1.16	5.30	20.99	26.10	33.77	36.40	29.32	0.16
22	0.0125	27.92	28.95	6.13	-1.22	5.50	20.34	24.97	32.08	34.47	27.95	0.15
22	0.0150	26.65	27.59	5.73	-1.28	5.71	19.57	23.87	30.50	32.67	26.68	0.14
22	0.0175	25.47	26.47	5.36	-1.34	5.92	18.87	22.99	29.09	31.01	25.50	0.13
22	0.0200	24.37	25.31	5.02	-1.39	6.13	18.15	22.06	27.82	29.58	24.39	0.12
22	0.0250	22.37	23.22	4.44	-1.50	6.54	16.92	20.41	25.40	26.95	22.39	0.11
22	0.0300	20.62	21.43	3.95	-1.59	6.92	15.78	18.86	23.30	24.61	20.63	0.10
22	0.0350	19.07	19.85	3.55	-1.67	7.27	14.71	17.53	21.53	22.57	19.09	0.09
22	0.0400	17.71	18.43	3.21	-1.73	7.57	13.77	16.34	19.92	20.87	17.73	0.08
23	0.0000	35.09	36.50	8.82	-0.85	4.44	23.50	30.50	40.50	44.50	35.12	0.19
23	0.0050	31.72	32.84	7.57	-0.98	4.79	22.17	27.86	36.67	39.91	31.75	0.16
23	0.0100	28.80	29.87	6.55	-1.12	5.18	20.67	25.55	33.25	35.92	28.83	0.14
23	0.0125	27.49	28.53	6.11	-1.18	5.38	19.92	24.48	31.65	34.07	27.51	0.13
23	0.0150	26.27	27.25	5.71	-1.24	5.59	19.16	23.47	30.14	32.35	26.29	0.12
23	0.0175	25.13	26.06	5.35	-1.30	5.80	18.54	22.49	28.72	30.76	25.15	0.12
23	0.0200	24.06	24.99	5.02	-1.36	6.01	17.87	21.69	27.40	29.25	24.08	0.11
23	0.0250	22.13	23.00	4.44	-1.47	6.43	16.68	20.02	25.22	26.66	22.15	0.10
23	0.0300	20.43	21.26	3.96	-1.56	6.84	15.58	18.64	23.17	24.47	20.44	0.09
23	0.0350	18.93	19.71	3.56	-1.65	7.22	14.59	17.37	21.35	22.48	18.94	0.08
23	0.0400	17.61	18.36	3.22	-1.73	7.56	13.63	16.19	19.86	20.74	17.62	0.07
24	0.0000	34.30	35.50	8.63	-0.79	4.29	23.50	29.50	40.50	44.50	34.30	0.22
24	0.0050	31.08	32.20	7.43	-0.93	4.62	21.69	27.26	35.85	39.20	31.07	0.19
24	0.0100	28.29	29.20	6.44	-1.06	4.99	20.16	25.00	32.66	35.31	28.28	0.17
24	0.0125	27.03	28.04	6.01	-1.12	5.18	19.55	24.00	31.13	33.58	27.02	0.16
24	0.0150	25.86	26.84	5.62	-1.18	5.39	18.85	23.10	29.69	31.94	25.85	0.15
24	0.0175	24.76	25.69	5.27	-1.24	5.59	18.15	22.18	28.33	30.40	24.75	0.14
24	0.0200	23.73	24.61	4.94	-1.30	5.80	17.51	21.31	27.05	28.93	23.72	0.13
24	0.0250	21.87	22.73	4.38	-1.41	6.22	16.39	19.82	24.84	26.32	21.85	0.12
24	0.0300	20.22	21.01	3.91	-1.51	6.63	15.37	18.39	22.93	24.20	20.21	0.10
24	0.0350	18.77	19.56	3.51	-1.61	7.03	14.40	17.21	21.18	22.31	18.75	0.09
24	0.0400	17.48	18.20	3.18	-1.69	7.40	13.56	16.12	19.64	20.61	17.46	0.09
25	0.0000	33.48	34.50	8.55	-0.78	4.22	22.50	28.50	39.50	43.50	33.50	0.19

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
25	0.0050	30.40	31.52	7.38	-0.91	4.53	21.08	26.52	35.35	38.60	30.43	0.17
25	0.0100	27.73	28.73	6.42	-1.04	4.88	19.58	24.47	31.98	34.66	27.75	0.15
25	0.0125	26.52	27.43	6.00	-1.10	5.06	18.87	23.46	30.60	33.02	26.54	0.14
25	0.0150	25.39	26.38	5.62	-1.16	5.25	18.36	22.56	29.26	31.48	25.41	0.13
25	0.0175	24.34	25.28	5.27	-1.22	5.44	17.73	21.77	27.98	30.00	24.35	0.12
25	0.0200	23.35	24.25	4.96	-1.28	5.64	17.15	20.93	26.75	28.61	23.36	0.11
25	0.0250	21.55	22.34	4.40	-1.38	6.04	16.08	19.45	24.51	26.09	21.56	0.10
25	0.0300	19.96	20.78	3.93	-1.48	6.44	15.06	18.14	22.66	23.91	19.97	0.09
25	0.0350	18.55	19.29	3.54	-1.58	6.83	14.19	16.98	21.01	22.09	18.55	0.08
25	0.0400	17.29	18.04	3.20	-1.66	7.20	13.29	15.86	19.51	20.48	17.30	0.07
26	0.0000	32.76	33.50	8.32	-0.73	4.11	22.50	28.50	38.50	42.50	32.68	0.22
26	0.0050	29.82	30.78	7.20	-0.86	4.39	20.64	26.00	34.73	37.96	29.75	0.19
26	0.0100	27.25	28.21	6.27	-0.98	4.70	19.31	24.05	31.37	34.01	27.19	0.17
26	0.0125	26.09	27.00	5.87	-1.04	4.86	18.70	23.13	29.98	32.43	26.04	0.16
26	0.0150	25.01	25.83	5.50	-1.10	5.04	18.05	22.24	28.73	30.96	24.95	0.15
26	0.0175	23.99	24.84	5.17	-1.16	5.21	17.41	21.37	27.57	29.58	23.94	0.14
26	0.0200	23.03	23.92	4.86	-1.21	5.39	16.85	20.62	26.41	28.24	22.98	0.13
26	0.0250	21.29	22.12	4.32	-1.32	5.76	15.86	19.22	24.28	25.83	21.25	0.12
26	0.0300	19.75	20.50	3.86	-1.42	6.12	14.84	17.91	22.36	23.69	19.71	0.10
26	0.0350	18.38	19.14	3.48	-1.51	6.48	14.04	16.79	20.78	21.85	18.35	0.09
26	0.0400	17.16	17.85	3.15	-1.59	6.82	13.17	15.74	19.38	20.29	17.13	0.09
27	0.0000	31.99	32.50	8.22	-0.69	4.02	21.50	27.50	37.50	41.50	31.84	0.19
27	0.0050	29.18	30.00	7.13	-0.82	4.28	20.08	25.41	34.05	37.32	29.05	0.17
27	0.0100	26.72	27.69	6.23	-0.94	4.57	18.64	23.37	30.96	33.64	26.61	0.15
27	0.0125	25.61	26.55	5.83	-1.00	4.73	18.10	22.58	29.53	31.96	25.51	0.14
27	0.0150	24.57	25.47	5.47	-1.06	4.90	17.60	21.78	28.19	30.41	24.47	0.13
27	0.0175	23.59	24.43	5.14	-1.12	5.07	17.02	20.98	27.04	29.09	23.49	0.12
27	0.0200	22.67	23.44	4.84	-1.17	5.25	16.45	20.23	25.99	27.85	22.58	0.11
27	0.0250	20.99	21.83	4.31	-1.28	5.61	15.47	18.92	23.99	25.57	20.91	0.10
27	0.0300	19.50	20.27	3.86	-1.38	5.98	14.59	17.67	22.18	23.52	19.43	0.09
27	0.0350	18.17	18.89	3.48	-1.48	6.34	13.71	16.55	20.56	21.70	18.11	0.08
27	0.0400	16.99	17.72	3.15	-1.57	6.70	13.00	15.58	19.16	20.09	16.93	0.07
28	0.0000	30.98	31.50	8.23	-0.68	3.91	20.50	26.50	36.50	40.50	31.02	0.16
28	0.0050	28.33	29.15	7.17	-0.80	4.14	19.11	24.55	33.25	36.47	28.36	0.14
28	0.0100	26.00	26.99	6.29	-0.92	4.41	17.72	22.66	30.31	32.91	26.03	0.12
28	0.0125	24.95	25.92	5.90	-0.98	4.55	17.32	21.85	28.93	31.29	24.97	0.11
28	0.0150	23.95	24.90	5.55	-1.04	4.70	16.84	21.18	27.62	29.86	23.98	0.11
28	0.0175	23.02	23.92	5.22	-1.09	4.85	16.31	20.43	26.50	28.59	23.05	0.10
28	0.0200	22.15	22.99	4.92	-1.14	5.01	15.80	19.69	25.49	27.40	22.17	0.09
28	0.0250	20.54	21.43	4.39	-1.25	5.33	14.85	18.38	23.62	25.21	20.56	0.08
28	0.0300	19.12	19.94	3.95	-1.34	5.66	14.04	17.27	21.87	23.22	19.14	0.08
28	0.0350	17.85	18.60	3.56	-1.44	5.99	13.19	16.18	20.30	21.46	17.86	0.07
28	0.0400	16.71	17.48	3.24	-1.52	6.32	12.53	15.26	18.91	19.91	16.72	0.06
29	0.0000	30.16	31.50	8.09	-0.65	3.83	19.50	25.50	35.50	39.50	30.16	0.20
29	0.0050	27.64	28.41	7.07	-0.77	4.05	18.53	23.84	32.44	35.67	27.64	0.17
29	0.0100	25.42	26.31	6.21	-0.88	4.29	17.43	22.17	29.69	32.29	25.43	0.15
29	0.0125	24.42	25.35	5.84	-0.94	4.42	16.84	21.33	28.40	30.76	24.42	0.14
29	0.0150	23.47	24.40	5.49	-0.99	4.56	16.26	20.54	27.19	29.30	23.47	0.13
29	0.0175	22.58	23.48	5.18	-1.05	4.70	15.82	19.96	26.04	28.08	22.58	0.12
29	0.0200	21.74	22.61	4.88	-1.10	4.85	15.46	19.31	24.99	26.94	21.73	0.12
29	0.0250	20.20	20.97	4.37	-1.20	5.15	14.55	18.04	23.22	24.84	20.19	0.10

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
29	0.0300	18.82	19.66	3.93	-1.30	5.47	13.68	16.94	21.59	22.93	18.82	0.09
29	0.0350	17.59	18.39	3.55	-1.39	5.78	13.04	15.93	20.09	21.23	17.59	0.08
29	0.0400	16.49	17.21	3.23	-1.47	6.10	12.31	14.99	18.73	19.72	16.49	0.08
30	0.0000	29.30	30.50	8.03	-0.61	3.75	18.50	24.50	34.50	38.50	29.29	0.22
30	0.0050	26.91	27.88	7.05	-0.73	3.95	17.67	23.06	31.68	34.97	26.90	0.19
30	0.0100	24.81	25.56	6.21	-0.85	4.17	16.77	21.56	29.10	31.81	24.79	0.17
30	0.0125	23.85	24.70	5.85	-0.90	4.30	16.27	20.82	27.89	30.37	23.83	0.16
30	0.0150	22.94	23.87	5.51	-0.96	4.43	15.75	20.08	26.75	29.00	22.93	0.15
30	0.0175	22.09	23.03	5.20	-1.01	4.56	15.24	19.36	25.66	27.71	22.07	0.14
30	0.0200	21.29	22.21	4.92	-1.07	4.70	14.79	18.79	24.63	26.50	21.27	0.13
30	0.0250	19.82	20.65	4.41	-1.17	5.00	14.12	17.68	22.79	24.45	19.80	0.12
30	0.0300	18.50	19.26	3.97	-1.26	5.30	13.32	16.57	21.26	22.66	18.48	0.10
30	0.0350	17.32	18.11	3.60	-1.36	5.61	12.59	15.66	19.85	21.04	17.30	0.09
30	0.0400	16.26	17.00	3.28	-1.44	5.92	12.06	14.77	18.56	19.56	16.24	0.08
31	0.0000	28.46	29.50	7.88	-0.58	3.63	18.50	23.50	33.50	37.50	28.41	0.22
31	0.0050	26.20	27.23	6.94	-0.69	3.80	16.76	22.17	30.85	34.14	26.16	0.19
31	0.0100	24.20	25.11	6.14	-0.80	4.00	16.06	20.90	28.44	31.14	24.16	0.17
31	0.0125	23.29	24.11	5.78	-0.85	4.11	15.67	20.22	27.32	29.74	23.25	0.16
31	0.0150	22.42	23.22	5.46	-0.91	4.22	15.22	19.55	26.24	28.44	22.39	0.15
31	0.0175	21.61	22.48	5.16	-0.96	4.35	14.76	18.88	25.21	27.21	21.58	0.14
31	0.0200	20.84	21.78	4.88	-1.01	4.47	14.30	18.23	24.23	26.05	20.82	0.13
31	0.0250	19.43	20.31	4.38	-1.10	4.73	13.55	17.24	22.43	24.05	19.41	0.11
31	0.0300	18.17	18.96	3.96	-1.20	5.01	12.93	16.20	20.88	22.33	18.14	0.10
31	0.0350	17.03	17.78	3.59	-1.29	5.29	12.24	15.23	19.56	20.78	17.01	0.09
31	0.0400	16.00	16.78	3.28	-1.37	5.58	11.61	14.48	18.34	19.35	15.99	0.08
32	0.0000	27.68	28.50	7.83	-0.53	3.49	17.50	23.50	32.50	36.50	27.58	0.19
32	0.0050	25.53	26.47	6.91	-0.64	3.64	16.40	21.74	30.00	33.37	25.44	0.17
32	0.0100	23.63	24.55	6.12	-0.75	3.81	15.31	20.16	27.77	30.61	23.55	0.15
32	0.0125	22.76	23.64	5.77	-0.80	3.91	14.92	19.63	26.74	29.29	22.68	0.14
32	0.0150	21.93	22.75	5.45	-0.85	4.01	14.63	19.05	25.76	28.06	21.87	0.13
32	0.0175	21.16	21.92	5.16	-0.90	4.12	14.35	18.45	24.80	26.90	21.09	0.12
32	0.0200	20.42	21.20	4.88	-0.95	4.22	13.97	17.86	23.88	25.81	20.36	0.12
32	0.0250	19.07	19.94	4.39	-1.04	4.45	13.20	16.75	22.18	23.79	19.02	0.10
32	0.0300	17.85	18.66	3.97	-1.13	4.69	12.45	15.91	20.62	22.00	17.81	0.09
32	0.0350	16.76	17.50	3.61	-1.21	4.94	11.99	15.00	19.23	20.50	16.72	0.09
32	0.0400	15.77	16.48	3.29	-1.30	5.20	11.38	14.17	18.08	19.16	15.73	0.08
33	0.0000	26.64	27.50	7.75	-0.52	3.54	16.50	22.50	31.50	35.50	26.71	0.17
33	0.0050	24.64	25.58	6.86	-0.63	3.68	15.68	20.87	29.15	32.52	24.70	0.15
33	0.0100	22.86	23.76	6.11	-0.74	3.85	14.81	19.35	27.04	29.82	22.91	0.13
33	0.0125	22.04	22.90	5.78	-0.79	3.95	14.33	18.87	26.06	28.58	22.09	0.13
33	0.0150	21.27	22.07	5.47	-0.85	4.05	13.89	18.39	25.09	27.40	21.31	0.12
33	0.0175	20.54	21.27	5.18	-0.90	4.16	13.59	17.82	24.16	26.27	20.58	0.11
33	0.0200	19.84	20.62	4.92	-0.95	4.27	13.34	17.29	23.28	25.21	19.88	0.11
33	0.0250	18.57	19.45	4.44	-1.05	4.51	12.74	16.26	21.63	23.28	18.60	0.10
33	0.0300	17.41	18.22	4.03	-1.14	4.76	12.07	15.37	20.14	21.63	17.44	0.09
33	0.0350	16.38	17.12	3.67	-1.23	5.02	11.41	14.63	18.89	20.20	16.40	0.08
33	0.0400	15.43	16.12	3.36	-1.31	5.28	11.06	13.85	17.79	18.91	15.46	0.08
34	0.0000	25.87	26.50	7.64	-0.48	3.44	15.50	21.50	31.50	34.50	25.82	0.21
34	0.0050	23.97	24.80	6.79	-0.59	3.56	14.92	20.21	28.29	31.69	23.93	0.18
34	0.0100	22.28	23.16	6.06	-0.70	3.71	14.31	18.96	26.31	29.20	22.24	0.16
34	0.0125	21.50	22.37	5.74	-0.75	3.79	13.95	18.33	25.39	28.03	21.46	0.16

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
34	0.0150	20.77	21.61	5.44	-0.80	3.88	13.60	17.72	24.51	26.93	20.73	0.15
34	0.0175	20.07	20.85	5.16	-0.85	3.97	13.21	17.25	23.68	25.87	20.03	0.14
34	0.0200	19.41	20.14	4.90	-0.89	4.06	12.83	16.85	22.88	24.87	19.37	0.13
34	0.0250	18.18	18.92	4.43	-0.99	4.26	12.19	15.93	21.33	23.03	18.15	0.12
34	0.0300	17.08	17.91	4.03	-1.08	4.48	11.75	15.04	19.92	21.38	17.05	0.11
34	0.0350	16.08	16.86	3.68	-1.16	4.70	11.16	14.20	18.65	19.89	16.06	0.10
34	0.0400	15.18	15.90	3.37	-1.24	4.93	10.62	13.58	17.48	18.64	15.15	0.09
35	0.0000	25.05	25.50	7.56	-0.42	3.32	14.50	20.50	30.50	33.50	24.98	0.19
35	0.0050	23.26	23.94	6.73	-0.52	3.42	13.99	19.39	27.92	30.85	23.20	0.17
35	0.0100	21.66	22.52	6.03	-0.63	3.54	13.50	18.28	25.58	28.49	21.60	0.15
35	0.0125	20.93	21.79	5.71	-0.68	3.61	13.27	17.71	24.70	27.40	20.87	0.14
35	0.0150	20.23	21.08	5.42	-0.73	3.69	13.04	17.15	23.87	26.38	20.17	0.13
35	0.0175	19.56	20.39	5.14	-0.78	3.77	12.71	16.61	23.09	25.40	19.51	0.13
35	0.0200	18.94	19.75	4.89	-0.82	3.85	12.42	16.17	22.34	24.44	18.89	0.12
35	0.0250	17.77	18.52	4.43	-0.92	4.04	11.79	15.47	20.95	22.68	17.73	0.11
35	0.0300	16.72	17.43	4.04	-1.00	4.23	11.20	14.63	19.60	21.08	16.68	0.10
35	0.0350	15.77	16.55	3.69	-1.09	4.44	10.80	13.87	18.39	19.66	15.73	0.09
35	0.0400	14.90	15.68	3.39	-1.17	4.66	10.38	13.15	17.27	18.38	14.86	0.08
36	0.0000	24.07	24.50	7.45	-0.40	3.31	14.50	19.50	29.50	32.50	24.11	0.19
36	0.0050	22.40	23.06	6.66	-0.50	3.39	13.53	18.52	26.92	30.00	22.44	0.17
36	0.0100	20.91	21.74	5.98	-0.60	3.50	12.63	17.53	24.81	27.77	20.94	0.15
36	0.0125	20.22	21.07	5.68	-0.65	3.57	12.43	17.02	24.00	26.74	20.25	0.14
36	0.0150	19.57	20.41	5.40	-0.70	3.64	12.23	16.52	23.22	25.76	19.59	0.13
36	0.0175	18.95	19.78	5.14	-0.75	3.71	12.03	16.02	22.48	24.81	18.97	0.13
36	0.0200	18.36	19.17	4.89	-0.79	3.79	11.84	15.54	21.78	23.90	18.38	0.12
36	0.0250	17.26	18.01	4.45	-0.88	3.96	11.32	14.85	20.45	22.21	17.28	0.11
36	0.0300	16.27	16.95	4.06	-0.97	4.14	10.79	14.17	19.17	20.70	16.28	0.10
36	0.0350	15.37	16.11	3.72	-1.05	4.34	10.27	13.46	18.01	19.33	15.38	0.09
36	0.0400	14.54	15.35	3.43	-1.13	4.54	9.88	12.80	16.95	18.08	14.55	0.08
37	0.0000	23.24	23.50	7.41	-0.39	3.24	13.50	18.50	28.50	31.50	23.28	0.17
37	0.0050	21.68	22.17	6.65	-0.49	3.32	12.83	17.67	26.32	29.15	21.71	0.15
37	0.0100	20.27	20.95	5.99	-0.59	3.42	12.12	16.87	24.32	27.04	20.30	0.14
37	0.0125	19.62	20.38	5.70	-0.64	3.48	11.77	16.42	23.38	26.07	19.65	0.13
37	0.0150	19.00	19.83	5.43	-0.69	3.54	11.40	15.98	22.56	25.14	19.03	0.12
37	0.0175	18.41	19.30	5.17	-0.73	3.61	11.24	15.55	21.87	24.27	18.44	0.12
37	0.0200	17.85	18.74	4.93	-0.78	3.68	11.07	15.12	21.20	23.43	17.88	0.11
37	0.0250	16.81	17.66	4.50	-0.86	3.84	10.75	14.28	19.96	21.82	16.83	0.10
37	0.0300	15.87	16.64	4.12	-0.95	4.01	10.32	13.66	18.82	20.37	15.89	0.09
37	0.0350	15.01	15.71	3.79	-1.03	4.19	9.84	13.11	17.72	19.06	15.03	0.08
37	0.0400	14.22	14.94	3.49	-1.11	4.38	9.40	12.48	16.71	17.86	14.24	0.08
38	0.0000	22.42	23.50	7.29	-0.36	3.18	12.50	18.50	27.50	31.50	22.39	0.18
38	0.0050	20.96	21.76	6.57	-0.46	3.24	12.11	16.76	25.55	28.41	20.93	0.16
38	0.0100	19.63	20.17	5.94	-0.55	3.33	11.66	16.06	23.76	26.31	19.61	0.14
38	0.0125	19.02	19.63	5.65	-0.60	3.38	11.42	15.73	22.89	25.39	19.00	0.14
38	0.0150	18.43	19.12	5.39	-0.64	3.43	11.15	15.40	22.07	24.51	18.42	0.13
38	0.0175	17.88	18.63	5.14	-0.69	3.49	10.89	15.01	21.29	23.68	17.86	0.12
38	0.0200	17.35	18.15	4.91	-0.73	3.55	10.63	14.63	20.62	22.89	17.33	0.12
38	0.0250	16.37	17.23	4.49	-0.82	3.68	10.08	13.88	19.45	21.43	16.35	0.11
38	0.0300	15.47	16.30	4.12	-0.90	3.83	9.75	13.16	18.37	20.04	15.46	0.10
38	0.0350	14.65	15.43	3.80	-0.97	3.99	9.49	12.59	17.38	18.79	14.64	0.09
38	0.0400	13.90	14.62	3.51	-1.05	4.15	9.12	12.14	16.43	17.65	13.89	0.08

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
39	0.0000	21.46	22.50	7.17	-0.31	3.15	11.50	17.50	26.50	30.50	21.55	0.18
39	0.0050	20.10	20.84	6.48	-0.41	3.19	11.18	16.24	24.66	27.57	20.18	0.16
39	0.0100	18.87	19.35	5.88	-0.51	3.26	10.87	15.22	22.98	25.56	18.95	0.14
39	0.0125	18.30	18.87	5.60	-0.55	3.30	10.71	14.92	22.17	24.70	18.38	0.14
39	0.0150	17.76	18.40	5.35	-0.60	3.35	10.55	14.63	21.40	23.87	17.83	0.13
39	0.0175	17.25	17.94	5.11	-0.64	3.40	10.33	14.35	20.67	23.09	17.31	0.12
39	0.0200	16.75	17.51	4.89	-0.69	3.46	10.11	14.04	20.02	22.34	16.81	0.12
39	0.0250	15.83	16.66	4.49	-0.77	3.58	9.67	13.35	18.92	20.95	15.89	0.11
39	0.0300	14.99	15.80	4.13	-0.85	3.71	9.22	12.72	17.91	19.61	15.04	0.10
39	0.0350	14.22	14.99	3.81	-0.93	3.85	8.81	12.10	16.98	18.42	14.27	0.09
39	0.0400	13.52	14.23	3.53	-1.00	4.01	8.60	11.61	16.09	17.32	13.56	0.08
40	0.0000	20.73	21.50	7.09	-0.25	3.09	11.50	16.50	25.50	29.50	20.73	0.17
40	0.0050	19.45	20.20	6.42	-0.35	3.13	10.89	15.65	23.87	27.26	19.45	0.16
40	0.0100	18.29	18.98	5.84	-0.44	3.18	10.31	14.83	22.36	25.21	18.29	0.14
40	0.0125	17.75	18.37	5.57	-0.49	3.21	10.03	14.42	21.64	24.28	17.76	0.14
40	0.0150	17.24	17.80	5.33	-0.53	3.25	9.74	14.01	20.94	23.35	17.24	0.13
40	0.0175	16.75	17.25	5.10	-0.58	3.30	9.60	13.62	20.28	22.48	16.75	0.12
40	0.0200	16.28	16.85	4.88	-0.62	3.34	9.48	13.34	19.62	21.78	16.29	0.12
40	0.0250	15.41	16.08	4.48	-0.70	3.45	9.25	12.88	18.41	20.46	15.41	0.11
40	0.0300	14.61	15.37	4.13	-0.78	3.56	8.96	12.37	17.43	19.26	14.62	0.10
40	0.0350	13.88	14.65	3.82	-0.86	3.69	8.60	11.80	16.55	18.16	13.88	0.09
40	0.0400	13.21	13.94	3.55	-0.93	3.82	8.27	11.28	15.74	17.11	13.21	0.09
41	0.0000	19.87	20.50	6.94	-0.24	3.08	10.50	15.50	24.50	28.50	19.87	0.19
41	0.0050	18.69	19.38	6.31	-0.34	3.10	10.23	14.88	23.01	26.41	18.68	0.17
41	0.0100	17.61	18.32	5.76	-0.43	3.15	9.92	14.26	21.62	24.51	17.61	0.16
41	0.0125	17.11	17.80	5.51	-0.47	3.18	9.74	13.93	20.96	23.63	17.11	0.15
41	0.0150	16.63	17.27	5.27	-0.51	3.21	9.55	13.60	20.31	22.79	16.63	0.14
41	0.0175	16.17	16.78	5.05	-0.56	3.25	9.33	13.26	19.69	21.98	16.17	0.14
41	0.0200	15.74	16.30	4.84	-0.60	3.29	9.14	12.93	19.09	21.21	15.73	0.13
41	0.0250	14.92	15.47	4.47	-0.68	3.38	8.74	12.27	17.94	19.96	14.92	0.12
41	0.0300	14.17	14.82	4.13	-0.75	3.49	8.33	11.79	16.94	18.82	14.17	0.11
41	0.0350	13.48	14.20	3.83	-0.83	3.60	8.10	11.41	16.11	17.78	13.48	0.10
41	0.0400	12.85	13.60	3.56	-0.90	3.72	7.93	11.00	15.35	16.82	12.84	0.09
42	0.0000	19.06	19.50	6.87	-0.20	3.02	9.50	14.50	23.50	27.50	19.04	0.19
42	0.0050	17.96	18.56	6.26	-0.30	3.04	9.28	13.99	22.17	25.59	17.95	0.17
42	0.0100	16.96	17.66	5.73	-0.39	3.07	9.06	13.50	20.93	23.82	16.95	0.15
42	0.0125	16.49	17.22	5.49	-0.43	3.09	8.96	13.27	20.33	23.00	16.48	0.15
42	0.0150	16.04	16.78	5.26	-0.47	3.12	8.86	12.98	19.74	22.21	16.03	0.14
42	0.0175	15.62	16.34	5.04	-0.51	3.15	8.76	12.70	19.16	21.46	15.61	0.13
42	0.0200	15.21	15.91	4.84	-0.55	3.19	8.66	12.42	18.62	20.75	15.20	0.13
42	0.0250	14.44	15.11	4.47	-0.63	3.27	8.32	11.88	17.58	19.45	14.43	0.12
42	0.0300	13.74	14.33	4.15	-0.71	3.36	8.00	11.34	16.61	18.37	13.73	0.11
42	0.0350	13.09	13.68	3.85	-0.78	3.46	7.68	10.82	15.72	17.38	13.08	0.10
42	0.0400	12.49	13.15	3.59	-0.85	3.57	7.37	10.48	14.94	16.48	12.49	0.09
43	0.0000	18.28	18.50	6.71	-0.18	2.99	9.50	13.50	22.50	26.50	18.23	0.15
43	0.0050	17.26	17.67	6.14	-0.27	2.99	9.05	13.05	21.28	24.74	17.21	0.14
43	0.0100	16.32	16.90	5.63	-0.35	3.01	8.61	12.63	20.16	23.13	16.28	0.13
43	0.0125	15.89	16.53	5.40	-0.39	3.03	8.38	12.43	19.63	22.37	15.85	0.12
43	0.0150	15.47	16.17	5.19	-0.43	3.05	8.16	12.23	19.12	21.64	15.43	0.12
43	0.0175	15.07	15.78	4.98	-0.47	3.08	7.93	12.03	18.61	20.94	15.04	0.11
43	0.0200	14.69	15.40	4.79	-0.51	3.11	7.82	11.84	18.10	20.26	14.66	0.11

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
43	0.0250	13.97	14.68	4.44	-0.59	3.18	7.66	11.43	17.14	19.00	13.94	0.10
43	0.0300	13.31	14.00	4.12	-0.66	3.26	7.51	10.97	16.24	17.91	13.28	0.09
43	0.0350	12.70	13.36	3.84	-0.73	3.35	7.37	10.50	15.41	16.98	12.67	0.09
43	0.0400	12.13	12.73	3.59	-0.80	3.44	7.17	10.04	14.64	16.12	12.11	0.08
44	0.0000	17.45	17.50	6.59	-0.11	2.99	8.50	13.50	21.50	25.50	17.44	0.16
44	0.0050	16.51	16.76	6.05	-0.19	2.98	8.32	12.85	20.39	23.93	16.50	0.14
44	0.0100	15.65	16.06	5.56	-0.28	2.98	8.15	12.23	19.35	22.46	15.64	0.13
44	0.0125	15.24	15.73	5.34	-0.32	2.99	8.03	11.92	18.87	21.77	15.24	0.13
44	0.0150	14.86	15.40	5.14	-0.36	3.01	7.90	11.59	18.40	21.09	14.85	0.12
44	0.0175	14.49	15.09	4.94	-0.40	3.02	7.77	11.28	17.94	20.44	14.48	0.12
44	0.0200	14.13	14.79	4.75	-0.44	3.05	7.62	11.07	17.51	19.82	14.13	0.11
44	0.0250	13.46	14.12	4.41	-0.52	3.10	7.32	10.75	16.62	18.63	13.46	0.10
44	0.0300	12.85	13.49	4.11	-0.59	3.16	7.03	10.45	15.79	17.52	12.85	0.10
44	0.0350	12.28	12.89	3.84	-0.66	3.24	6.73	10.15	15.02	16.55	12.28	0.09
44	0.0400	11.75	12.33	3.59	-0.73	3.32	6.49	9.75	14.30	15.74	11.75	0.08
45	0.0000	16.62	16.50	6.49	-0.08	2.93	7.50	12.50	20.50	24.50	16.61	0.16
45	0.0050	15.76	15.84	5.97	-0.17	2.91	7.36	12.05	19.49	23.06	15.74	0.15
45	0.0100	14.97	15.22	5.51	-0.25	2.92	7.23	11.62	18.54	21.73	14.95	0.14
45	0.0125	14.60	14.92	5.30	-0.29	2.92	7.16	11.37	18.10	21.07	14.58	0.13
45	0.0150	14.24	14.63	5.10	-0.33	2.93	7.09	11.14	17.67	20.45	14.22	0.13
45	0.0175	13.90	14.35	4.91	-0.37	2.95	7.03	10.91	17.25	19.85	13.88	0.12
45	0.0200	13.57	14.07	4.73	-0.41	2.97	6.97	10.69	16.85	19.27	13.55	0.12
45	0.0250	12.95	13.55	4.40	-0.48	3.01	6.84	10.22	16.08	18.18	12.93	0.11
45	0.0300	12.38	13.06	4.11	-0.55	3.07	6.72	9.78	15.37	17.16	12.36	0.10
45	0.0350	11.85	12.52	3.84	-0.62	3.13	6.51	9.49	14.64	16.22	11.83	0.10
45	0.0400	11.35	12.02	3.60	-0.68	3.21	6.30	9.25	13.96	15.36	11.34	0.09
46	0.0000	15.80	16.50	6.34	-0.01	2.91	7.50	11.50	19.50	23.50	15.84	0.15
46	0.0050	15.01	15.07	5.85	-0.10	2.88	7.22	11.18	18.58	22.17	15.04	0.13
46	0.0100	14.28	14.36	5.42	-0.18	2.86	6.94	10.87	17.72	20.95	14.31	0.12
46	0.0125	13.94	14.10	5.22	-0.22	2.86	6.80	10.71	17.32	20.38	13.97	0.12
46	0.0150	13.61	13.84	5.03	-0.25	2.86	6.67	10.54	16.92	19.82	13.64	0.11
46	0.0175	13.30	13.59	4.85	-0.29	2.87	6.53	10.37	16.54	19.25	13.33	0.11
46	0.0200	12.99	13.34	4.68	-0.33	2.88	6.40	10.18	16.17	18.71	13.02	0.11
46	0.0250	12.42	12.88	4.36	-0.40	2.91	6.10	9.81	15.47	17.68	12.45	0.10
46	0.0300	11.89	12.43	4.08	-0.47	2.95	5.91	9.43	14.82	16.72	11.91	0.09
46	0.0350	11.40	12.01	3.82	-0.53	2.99	5.82	9.06	14.20	15.84	11.42	0.09
46	0.0400	10.94	11.57	3.59	-0.60	3.05	5.73	8.71	13.57	15.04	10.96	0.08
47	0.0000	14.99	15.50	6.22	0.05	2.92	6.50	10.50	19.50	22.50	15.04	0.14
47	0.0050	14.27	14.73	5.75	-0.04	2.87	6.40	10.23	18.23	21.28	14.31	0.13
47	0.0100	13.60	14.00	5.34	-0.12	2.84	6.29	9.97	17.06	20.16	13.65	0.12
47	0.0125	13.29	13.65	5.15	-0.15	2.83	6.24	9.84	16.53	19.63	13.33	0.12
47	0.0150	12.99	13.29	4.96	-0.19	2.83	6.19	9.72	16.17	19.12	13.03	0.11
47	0.0175	12.70	12.95	4.79	-0.23	2.82	6.14	9.60	15.82	18.63	12.74	0.11
47	0.0200	12.42	12.63	4.63	-0.27	2.83	6.10	9.48	15.49	18.13	12.46	0.11
47	0.0250	11.89	12.19	4.33	-0.34	2.84	6.00	9.25	14.85	17.17	11.93	0.10
47	0.0300	11.40	11.79	4.06	-0.40	2.87	5.86	8.98	14.25	16.29	11.43	0.09
47	0.0350	10.94	11.41	3.81	-0.47	2.90	5.64	8.68	13.68	15.47	10.98	0.09
47	0.0400	10.52	11.06	3.59	-0.53	2.95	5.46	8.38	13.15	14.69	10.55	0.08
48	0.0000	14.30	14.50	6.08	0.05	2.91	6.50	10.50	18.50	21.50	14.24	0.15
48	0.0050	13.64	13.96	5.64	-0.03	2.86	6.18	10.12	17.58	20.39	13.58	0.14
48	0.0100	13.02	13.44	5.25	-0.10	2.83	5.88	9.75	16.70	19.35	12.97	0.13

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
48	0.0125	12.73	13.18	5.06	-0.14	2.82	5.74	9.56	16.27	18.87	12.68	0.12
48	0.0150	12.45	12.92	4.89	-0.18	2.81	5.57	9.34	15.85	18.40	12.40	0.12
48	0.0175	12.18	12.66	4.73	-0.21	2.81	5.42	9.15	15.44	17.94	12.14	0.11
48	0.0200	11.92	12.40	4.58	-0.25	2.81	5.28	8.95	15.05	17.51	11.88	0.11
48	0.0250	11.43	11.91	4.29	-0.32	2.82	5.14	8.54	14.29	16.68	11.39	0.10
48	0.0300	10.97	11.42	4.03	-0.38	2.84	5.07	8.28	13.66	15.91	10.94	0.10
48	0.0350	10.54	10.95	3.79	-0.45	2.87	5.01	8.10	13.14	15.13	10.52	0.09
48	0.0400	10.15	10.50	3.58	-0.51	2.91	4.94	7.93	12.66	14.42	10.12	0.09
49	0.0000	13.49	13.50	5.88	0.14	2.93	5.50	9.50	17.50	20.50	13.49	0.16
49	0.0050	12.89	13.05	5.47	0.06	2.86	5.42	9.28	16.69	19.49	12.89	0.15
49	0.0100	12.33	12.63	5.10	-0.02	2.81	5.35	9.06	15.92	18.54	12.33	0.14
49	0.0125	12.07	12.41	4.93	-0.06	2.79	5.31	8.96	15.54	18.10	12.07	0.13
49	0.0150	11.82	12.19	4.77	-0.09	2.78	5.28	8.84	15.18	17.67	11.81	0.13
49	0.0175	11.57	11.97	4.62	-0.13	2.77	5.24	8.71	14.80	17.25	11.57	0.12
49	0.0200	11.34	11.75	4.47	-0.16	2.76	5.21	8.58	14.45	16.85	11.33	0.12
49	0.0250	10.89	11.32	4.20	-0.23	2.75	5.14	8.31	13.77	16.08	10.89	0.11
49	0.0300	10.47	10.90	3.95	-0.29	2.76	5.07	8.04	13.13	15.37	10.47	0.10
49	0.0350	10.08	10.50	3.73	-0.35	2.77	4.96	7.78	12.59	14.70	10.08	0.10
49	0.0400	9.72	10.12	3.53	-0.41	2.80	4.82	7.50	12.14	14.02	9.71	0.09
50	0.0000	12.76	12.50	5.70	0.19	2.97	5.50	8.50	16.50	20.50	12.73	0.17
50	0.0050	12.21	12.12	5.31	0.11	2.90	5.28	8.32	15.80	19.06	12.18	0.16
50	0.0100	11.70	11.75	4.97	0.03	2.84	5.08	8.15	15.13	17.78	11.68	0.15
50	0.0125	11.46	11.58	4.81	0.00	2.82	4.98	8.07	14.80	17.32	11.44	0.15
50	0.0150	11.23	11.40	4.65	-0.04	2.80	4.88	7.98	14.48	16.92	11.20	0.14
50	0.0175	11.00	11.24	4.51	-0.07	2.78	4.77	7.90	14.16	16.54	10.98	0.14
50	0.0200	10.79	11.07	4.37	-0.10	2.77	4.68	7.82	13.86	16.17	10.77	0.13
50	0.0250	10.38	10.75	4.12	-0.17	2.75	4.49	7.66	13.26	15.47	10.36	0.13
50	0.0300	9.99	10.42	3.88	-0.23	2.74	4.28	7.51	12.69	14.82	9.98	0.12
50	0.0350	9.63	10.06	3.67	-0.29	2.75	4.17	7.37	12.14	14.20	9.62	0.11
50	0.0400	9.30	9.72	3.47	-0.35	2.76	4.12	7.22	11.62	13.58	9.28	0.11
51	0.0000	12.02	11.50	5.57	0.24	2.99	4.50	8.50	15.50	19.50	12.01	0.15
51	0.0050	11.53	11.18	5.21	0.16	2.90	4.45	8.20	14.92	18.34	11.51	0.14
51	0.0100	11.07	10.87	4.88	0.09	2.83	4.40	7.92	14.36	17.26	11.05	0.13
51	0.0125	10.85	10.71	4.73	0.06	2.81	4.38	7.77	14.10	16.72	10.83	0.13
51	0.0150	10.64	10.57	4.59	0.02	2.78	4.35	7.63	13.84	16.24	10.62	0.12
51	0.0175	10.43	10.42	4.45	-0.01	2.76	4.33	7.48	13.56	15.82	10.42	0.12
51	0.0200	10.24	10.28	4.32	-0.04	2.74	4.30	7.34	13.29	15.49	10.22	0.11
51	0.0250	9.86	10.01	4.07	-0.11	2.71	4.26	7.07	12.77	14.85	9.85	0.11
51	0.0300	9.51	9.75	3.85	-0.17	2.70	4.21	6.78	12.26	14.25	9.50	0.10
51	0.0350	9.18	9.49	3.65	-0.22	2.69	4.17	6.61	11.77	13.68	9.17	0.10
51	0.0400	8.88	9.25	3.46	-0.28	2.69	4.12	6.49	11.32	13.15	8.86	0.09
52	0.0000	11.26	11.50	5.39	0.29	2.94	4.50	7.50	14.50	18.50	11.30	0.14
52	0.0050	10.82	10.96	5.06	0.22	2.86	4.36	7.36	13.99	17.57	10.86	0.13
52	0.0100	10.40	10.46	4.76	0.15	2.79	4.23	7.23	13.50	16.69	10.44	0.13
52	0.0125	10.21	10.20	4.61	0.12	2.76	4.17	7.16	13.27	16.26	10.24	0.12
52	0.0150	10.02	9.96	4.48	0.09	2.73	4.11	7.09	13.04	15.84	10.05	0.12
52	0.0175	9.83	9.70	4.35	0.06	2.71	4.05	7.03	12.82	15.43	9.86	0.11
52	0.0200	9.66	9.48	4.23	0.02	2.69	3.97	6.95	12.60	15.04	9.69	0.11
52	0.0250	9.32	9.25	4.00	-0.03	2.66	3.82	6.76	12.19	14.28	9.35	0.10
52	0.0300	9.00	9.02	3.79	-0.09	2.63	3.70	6.57	11.74	13.66	9.03	0.10
52	0.0350	8.70	8.81	3.60	-0.15	2.62	3.54	6.39	11.31	13.14	8.73	0.09

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
52	0.0400	8.42	8.60	3.42	-0.20	2.61	3.40	6.21	10.90	12.66	8.44	0.09
53	0.0000	10.57	10.50	5.21	0.35	3.01	3.50	6.50	13.50	17.50	10.59	0.16
53	0.0050	10.17	10.20	4.90	0.28	2.92	3.47	6.40	13.05	16.67	10.19	0.15
53	0.0100	9.80	9.90	4.62	0.21	2.84	3.44	6.29	12.63	15.89	9.82	0.14
53	0.0125	9.62	9.75	4.48	0.18	2.81	3.42	6.24	12.43	15.51	9.64	0.13
53	0.0150	9.45	9.60	4.36	0.15	2.78	3.41	6.19	12.23	15.12	9.46	0.13
53	0.0175	9.28	9.45	4.24	0.12	2.75	3.39	6.14	12.03	14.76	9.30	0.13
53	0.0200	9.12	9.30	4.12	0.09	2.72	3.38	6.10	11.84	14.38	9.14	0.12
53	0.0250	8.81	9.00	3.91	0.03	2.68	3.35	6.00	11.48	13.71	8.83	0.12
53	0.0300	8.52	8.70	3.71	-0.03	2.65	3.32	5.91	11.13	13.07	8.54	0.11
53	0.0350	8.25	8.40	3.53	-0.08	2.63	3.29	5.82	10.80	12.59	8.27	0.10
53	0.0400	8.00	8.11	3.36	-0.13	2.61	3.27	5.73	10.48	12.14	8.01	0.10
54	0.0000	9.86	9.50	5.06	0.43	3.05	3.50	6.50	13.50	16.50	9.91	0.14
54	0.0050	9.51	9.28	4.77	0.36	2.95	3.47	6.34	12.81	15.84	9.55	0.13
54	0.0100	9.17	9.06	4.51	0.30	2.86	3.44	6.18	12.17	15.22	9.21	0.12
54	0.0125	9.02	8.96	4.38	0.27	2.82	3.42	6.10	11.85	14.89	9.05	0.12
54	0.0150	8.86	8.86	4.27	0.24	2.78	3.41	6.00	11.55	14.58	8.89	0.12
54	0.0175	8.71	8.76	4.15	0.21	2.75	3.39	5.91	11.26	14.27	8.74	0.11
54	0.0200	8.57	8.66	4.04	0.18	2.72	3.38	5.82	11.07	13.96	8.60	0.11
54	0.0250	8.29	8.47	3.84	0.12	2.67	3.35	5.64	10.75	13.35	8.32	0.10
54	0.0300	8.03	8.21	3.65	0.07	2.63	3.32	5.48	10.45	12.79	8.06	0.10
54	0.0350	7.79	8.00	3.48	0.02	2.59	3.26	5.30	10.16	12.25	7.81	0.10
54	0.0400	7.56	7.77	3.32	-0.03	2.57	3.17	5.13	9.88	11.74	7.58	0.09
55	0.0000	9.25	8.50	4.91	0.51	3.13	3.50	5.50	12.50	15.50	9.26	0.16
55	0.0050	8.92	8.32	4.64	0.45	3.02	3.34	5.42	12.06	14.92	8.93	0.15
55	0.0100	8.62	8.15	4.39	0.39	2.92	3.19	5.35	11.65	14.36	8.63	0.14
55	0.0125	8.48	8.07	4.27	0.36	2.88	3.11	5.31	11.43	14.10	8.49	0.13
55	0.0150	8.34	7.98	4.16	0.33	2.83	3.05	5.28	11.22	13.84	8.35	0.13
55	0.0175	8.20	7.90	4.05	0.30	2.80	2.98	5.24	11.02	13.59	8.21	0.13
55	0.0200	8.07	7.82	3.95	0.27	2.76	2.92	5.21	10.81	13.34	8.08	0.12
55	0.0250	7.82	7.66	3.76	0.22	2.70	2.80	5.14	10.40	12.88	7.83	0.12
55	0.0300	7.58	7.51	3.58	0.17	2.64	2.68	5.07	10.03	12.40	7.59	0.11
55	0.0350	7.36	7.37	3.42	0.12	2.60	2.55	5.01	9.65	11.92	7.37	0.11
55	0.0400	7.15	7.22	3.26	0.07	2.56	2.44	4.94	9.30	11.47	7.16	0.10
56	0.0000	8.56	8.50	4.76	0.60	3.21	2.50	5.50	11.50	15.50	8.60	0.15
56	0.0050	8.28	8.13	4.51	0.54	3.10	2.48	5.28	11.16	14.20	8.31	0.14
56	0.0100	8.01	7.79	4.27	0.48	3.00	2.47	5.08	10.83	13.50	8.05	0.13
56	0.0125	7.88	7.63	4.16	0.45	2.95	2.46	4.98	10.67	13.27	7.92	0.13
56	0.0150	7.76	7.47	4.06	0.42	2.91	2.45	4.88	10.50	13.04	7.79	0.12
56	0.0175	7.64	7.31	3.96	0.40	2.86	2.44	4.78	10.33	12.82	7.67	0.12
56	0.0200	7.52	7.13	3.86	0.37	2.83	2.44	4.69	10.17	12.60	7.56	0.12
56	0.0250	7.30	6.84	3.68	0.32	2.75	2.42	4.52	9.85	12.19	7.33	0.11
56	0.0300	7.09	6.72	3.51	0.27	2.69	2.41	4.33	9.52	11.79	7.12	0.11
56	0.0350	6.89	6.61	3.36	0.22	2.64	2.39	4.17	9.21	11.41	6.92	0.10
56	0.0400	6.70	6.49	3.21	0.17	2.59	2.38	4.12	8.90	11.06	6.73	0.10
57	0.0000	7.95	7.50	4.60	0.70	3.46	2.50	4.50	10.50	14.50	7.96	0.13
57	0.0050	7.70	7.36	4.37	0.64	3.31	2.48	4.45	10.23	13.80	7.70	0.12
57	0.0100	7.46	7.23	4.15	0.58	3.18	2.47	4.40	9.97	13.14	7.46	0.12
57	0.0125	7.35	7.16	4.04	0.55	3.12	2.46	4.38	9.84	12.83	7.35	0.12
57	0.0150	7.24	7.09	3.95	0.52	3.07	2.45	4.35	9.72	12.53	7.24	0.11
57	0.0175	7.13	7.03	3.85	0.50	3.02	2.44	4.33	9.60	12.21	7.13	0.11

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
57	0.0200	7.02	6.93	3.76	0.47	2.97	2.44	4.30	9.48	11.90	7.03	0.11
57	0.0250	6.82	6.78	3.59	0.42	2.87	2.42	4.26	9.25	11.48	6.83	0.10
57	0.0300	6.63	6.61	3.43	0.37	2.79	2.41	4.21	9.02	11.13	6.64	0.10
57	0.0350	6.46	6.44	3.29	0.32	2.72	2.39	4.17	8.81	10.80	6.46	0.09
57	0.0400	6.29	6.28	3.15	0.27	2.66	2.38	4.12	8.54	10.48	6.29	0.09
58	0.0000	7.35	6.50	4.48	0.76	3.49	1.50	3.50	10.50	13.50	7.34	0.13
58	0.0050	7.12	6.40	4.26	0.70	3.35	1.49	3.47	9.95	13.01	7.12	0.13
58	0.0100	6.91	6.29	4.05	0.65	3.23	1.49	3.44	9.44	12.55	6.90	0.12
58	0.0125	6.81	6.24	3.96	0.62	3.17	1.48	3.42	9.19	12.31	6.80	0.12
58	0.0150	6.71	6.19	3.87	0.59	3.11	1.48	3.41	8.95	12.08	6.70	0.11
58	0.0175	6.62	6.14	3.78	0.57	3.06	1.48	3.39	8.76	11.86	6.61	0.11
58	0.0200	6.52	6.10	3.69	0.54	3.01	1.48	3.38	8.66	11.63	6.52	0.11
58	0.0250	6.34	6.00	3.53	0.50	2.92	1.47	3.35	8.47	11.20	6.34	0.10
58	0.0300	6.18	5.91	3.38	0.45	2.83	1.46	3.32	8.28	10.77	6.17	0.10
58	0.0350	6.02	5.82	3.24	0.40	2.76	1.46	3.29	8.10	10.35	6.01	0.10
58	0.0400	5.86	5.73	3.11	0.36	2.69	1.45	3.27	7.93	9.97	5.86	0.09
59	0.0000	6.74	6.50	4.31	0.84	3.68	1.50	3.50	9.50	12.50	6.74	0.13
59	0.0050	6.54	6.23	4.11	0.79	3.53	1.49	3.47	9.20	12.12	6.54	0.12
59	0.0100	6.36	5.99	3.92	0.74	3.39	1.49	3.44	8.92	11.75	6.35	0.12
59	0.0125	6.27	5.87	3.83	0.71	3.33	1.48	3.42	8.78	11.58	6.26	0.11
59	0.0150	6.18	5.74	3.75	0.69	3.27	1.48	3.41	8.63	11.40	6.18	0.11
59	0.0175	6.09	5.63	3.66	0.66	3.21	1.48	3.39	8.48	11.24	6.09	0.11
59	0.0200	6.01	5.51	3.59	0.64	3.16	1.48	3.38	8.33	11.06	6.01	0.11
59	0.0250	5.85	5.28	3.44	0.59	3.05	1.47	3.35	8.07	10.68	5.85	0.10
59	0.0300	5.70	5.08	3.30	0.55	2.96	1.46	3.32	7.78	10.32	5.70	0.10
59	0.0350	5.56	5.01	3.17	0.50	2.88	1.46	3.29	7.48	9.97	5.56	0.09
59	0.0400	5.42	4.94	3.05	0.46	2.80	1.45	3.27	7.22	9.62	5.43	0.09
60	0.0000	6.19	5.50	4.14	0.89	3.71	1.50	2.50	8.50	11.50	6.20	0.12
60	0.0050	6.01	5.42	3.96	0.84	3.56	1.49	2.48	8.32	11.18	6.03	0.12
60	0.0100	5.85	5.35	3.79	0.79	3.43	1.49	2.47	8.15	10.87	5.86	0.11
60	0.0125	5.77	5.31	3.71	0.77	3.36	1.48	2.46	8.07	10.71	5.78	0.11
60	0.0150	5.70	5.28	3.63	0.75	3.30	1.48	2.45	7.98	10.57	5.71	0.11
60	0.0175	5.62	5.24	3.55	0.72	3.24	1.48	2.44	7.90	10.42	5.63	0.10
60	0.0200	5.55	5.21	3.48	0.70	3.19	1.48	2.44	7.82	10.28	5.56	0.10
60	0.0250	5.41	5.14	3.34	0.66	3.08	1.47	2.42	7.66	10.01	5.42	0.10
60	0.0300	5.28	5.07	3.21	0.62	2.99	1.46	2.41	7.51	9.75	5.29	0.10
60	0.0350	5.15	5.01	3.09	0.57	2.90	1.46	2.39	7.32	9.49	5.16	0.09
60	0.0400	5.03	4.91	2.98	0.54	2.82	1.45	2.38	7.12	9.25	5.04	0.09
61	0.0000	5.74	5.50	4.01	0.98	3.96	1.50	2.50	8.50	11.50	5.76	0.13
61	0.0050	5.59	5.12	3.84	0.93	3.80	1.49	2.48	7.87	11.08	5.60	0.12
61	0.0100	5.44	4.78	3.68	0.89	3.65	1.49	2.47	7.29	10.68	5.45	0.12
61	0.0125	5.37	4.62	3.60	0.86	3.58	1.48	2.46	7.16	10.49	5.38	0.11
61	0.0150	5.30	4.47	3.53	0.84	3.52	1.48	2.45	7.09	10.29	5.31	0.11
61	0.0175	5.24	4.33	3.46	0.82	3.45	1.48	2.44	7.03	10.08	5.25	0.11
61	0.0200	5.17	4.30	3.39	0.79	3.39	1.48	2.44	6.97	9.90	5.18	0.11
61	0.0250	5.05	4.26	3.26	0.75	3.28	1.47	2.42	6.84	9.51	5.06	0.10
61	0.0300	4.93	4.21	3.14	0.71	3.17	1.46	2.41	6.72	9.13	4.94	0.10
61	0.0350	4.81	4.17	3.03	0.67	3.07	1.46	2.39	6.61	8.81	4.82	0.10
61	0.0400	4.71	4.12	2.92	0.63	2.98	1.45	2.38	6.49	8.60	4.72	0.09
62	0.0000	5.35	4.50	3.89	1.04	4.04	1.50	2.50	7.50	10.50	5.31	0.12
62	0.0050	5.21	4.45	3.73	0.99	3.88	1.42	2.48	7.36	10.23	5.17	0.11

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active
Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
62	0.0100	5.08	4.40	3.58	0.95	3.73	1.34	2.47	7.23	9.97	5.04	0.11
62	0.0125	5.02	4.38	3.51	0.92	3.67	1.31	2.46	7.16	9.84	4.98	0.11
62	0.0150	4.96	4.35	3.44	0.90	3.60	1.27	2.45	7.09	9.72	4.92	0.11
62	0.0175	4.90	4.33	3.37	0.88	3.54	1.24	2.44	7.03	9.60	4.86	0.10
62	0.0200	4.84	4.30	3.31	0.86	3.47	1.21	2.44	6.97	9.48	4.80	0.10
62	0.0250	4.72	4.26	3.19	0.82	3.36	1.15	2.42	6.84	9.25	4.69	0.10
62	0.0300	4.62	4.21	3.07	0.78	3.25	1.10	2.41	6.70	9.02	4.58	0.10
62	0.0350	4.52	4.17	2.97	0.74	3.15	1.05	2.39	6.53	8.80	4.48	0.09
62	0.0400	4.42	4.12	2.87	0.70	3.06	1.00	2.38	6.38	8.51	4.39	0.09
63	0.0000	4.97	4.50	3.72	1.07	4.13	0.50	2.50	7.50	10.50	4.96	0.13
63	0.0050	4.85	4.41	3.57	1.03	3.97	0.50	2.43	7.08	10.06	4.84	0.12
63	0.0100	4.74	4.32	3.43	0.98	3.82	0.50	2.35	6.69	9.64	4.72	0.12
63	0.0125	4.68	4.28	3.37	0.96	3.75	0.50	2.32	6.52	9.44	4.66	0.12
63	0.0150	4.62	4.24	3.31	0.94	3.68	0.50	2.29	6.35	9.23	4.61	0.12
63	0.0175	4.57	4.20	3.25	0.92	3.62	0.50	2.25	6.18	9.05	4.56	0.11
63	0.0200	4.52	4.16	3.19	0.90	3.56	0.50	2.22	6.10	8.87	4.50	0.11
63	0.0250	4.42	4.06	3.08	0.86	3.44	0.49	2.16	6.00	8.49	4.40	0.11
63	0.0300	4.32	3.98	2.97	0.82	3.33	0.49	2.10	5.91	8.28	4.31	0.10
63	0.0350	4.23	3.89	2.87	0.79	3.23	0.49	2.03	5.82	8.10	4.22	0.10
63	0.0400	4.14	3.78	2.78	0.75	3.13	0.49	1.97	5.73	7.93	4.13	0.10
64	0.0000	4.60	3.50	3.59	1.17	4.36	0.50	1.50	6.50	9.50	4.64	0.11
64	0.0050	4.50	3.47	3.46	1.12	4.19	0.50	1.49	6.40	9.28	4.53	0.11
64	0.0100	4.39	3.44	3.33	1.08	4.04	0.50	1.49	6.29	9.06	4.43	0.10
64	0.0125	4.34	3.42	3.27	1.06	3.97	0.50	1.48	6.24	8.96	4.38	0.10
64	0.0150	4.30	3.41	3.21	1.03	3.90	0.50	1.48	6.19	8.86	4.33	0.10
64	0.0175	4.25	3.39	3.16	1.01	3.83	0.50	1.48	6.14	8.76	4.28	0.10
64	0.0200	4.20	3.38	3.10	0.99	3.77	0.50	1.48	6.10	8.66	4.24	0.10
64	0.0250	4.11	3.35	3.00	0.96	3.64	0.49	1.47	6.00	8.47	4.14	0.10
64	0.0300	4.03	3.32	2.90	0.92	3.53	0.49	1.46	5.91	8.28	4.06	0.09
64	0.0350	3.95	3.29	2.81	0.88	3.42	0.49	1.46	5.82	8.10	3.98	0.09
64	0.0400	3.87	3.27	2.72	0.85	3.32	0.49	1.45	5.73	7.87	3.90	0.09
65	0.0000	4.37	3.50	3.48	1.17	4.28	0.50	1.50	6.50	9.50	4.38	0.12
65	0.0050	4.27	3.47	3.36	1.13	4.13	0.50	1.49	6.34	9.22	4.29	0.12
65	0.0100	4.18	3.44	3.24	1.09	3.98	0.50	1.49	6.18	8.95	4.19	0.12
65	0.0125	4.14	3.42	3.18	1.07	3.92	0.50	1.48	6.11	8.82	4.15	0.11
65	0.0150	4.09	3.41	3.13	1.05	3.85	0.50	1.48	6.04	8.68	4.10	0.11
65	0.0175	4.05	3.39	3.08	1.03	3.79	0.50	1.48	5.96	8.56	4.06	0.11
65	0.0200	4.01	3.38	3.03	1.01	3.73	0.50	1.48	5.89	8.42	4.02	0.11
65	0.0250	3.93	3.35	2.93	0.97	3.61	0.49	1.47	5.75	8.17	3.94	0.11
65	0.0300	3.85	3.32	2.84	0.94	3.50	0.49	1.46	5.60	7.92	3.86	0.10
65	0.0350	3.78	3.29	2.75	0.90	3.40	0.49	1.46	5.46	7.66	3.78	0.10
65	0.0400	3.70	3.27	2.67	0.87	3.30	0.49	1.45	5.29	7.42	3.71	0.10
66	0.0000	4.19	3.50	3.39	1.19	4.31	0.50	1.50	5.50	8.50	4.14	0.12
66	0.0050	4.10	3.47	3.27	1.15	4.16	0.50	1.49	5.42	8.32	4.05	0.11
66	0.0100	4.01	3.44	3.16	1.11	4.02	0.50	1.49	5.35	8.15	3.97	0.11
66	0.0125	3.97	3.42	3.11	1.09	3.96	0.50	1.48	5.31	8.07	3.92	0.11
66	0.0150	3.93	3.41	3.06	1.08	3.89	0.50	1.48	5.28	7.98	3.89	0.11
66	0.0175	3.89	3.39	3.01	1.06	3.83	0.50	1.48	5.24	7.90	3.85	0.11
66	0.0200	3.85	3.38	2.96	1.04	3.77	0.50	1.48	5.21	7.82	3.81	0.10
66	0.0250	3.78	3.35	2.87	1.00	3.66	0.49	1.47	5.14	7.66	3.74	0.10
66	0.0300	3.71	3.32	2.78	0.97	3.55	0.49	1.46	5.07	7.51	3.66	0.10

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
66	0.0350	3.64	3.29	2.70	0.94	3.45	0.49	1.46	5.01	7.37	3.60	0.10
66	0.0400	3.57	3.27	2.62	0.90	3.36	0.49	1.45	4.94	7.22	3.53	0.09
67	0.0000	3.95	3.50	3.25	1.23	4.38	0.50	1.50	5.50	8.50	3.96	0.14
67	0.0050	3.87	3.42	3.15	1.19	4.24	0.50	1.49	5.42	8.32	3.88	0.13
67	0.0100	3.80	3.35	3.05	1.15	4.10	0.50	1.49	5.35	8.15	3.80	0.13
67	0.0125	3.76	3.32	3.00	1.13	4.04	0.50	1.48	5.31	8.07	3.76	0.13
67	0.0150	3.73	3.28	2.96	1.11	3.98	0.50	1.48	5.28	7.98	3.73	0.13
67	0.0175	3.69	3.25	2.91	1.10	3.92	0.50	1.48	5.24	7.90	3.69	0.13
67	0.0200	3.66	3.21	2.87	1.08	3.86	0.50	1.48	5.21	7.82	3.66	0.12
67	0.0250	3.59	3.15	2.78	1.04	3.74	0.49	1.47	5.14	7.66	3.59	0.12
67	0.0300	3.52	3.08	2.71	1.01	3.64	0.49	1.46	5.07	7.51	3.52	0.12
67	0.0350	3.46	3.02	2.63	0.98	3.53	0.49	1.46	5.01	7.37	3.46	0.12
67	0.0400	3.40	2.95	2.56	0.95	3.44	0.49	1.45	4.94	7.22	3.40	0.11
68	0.0000	3.76	2.50	3.15	1.28	4.53	0.50	1.50	5.50	8.50	3.79	0.13
68	0.0050	3.68	2.48	3.05	1.24	4.39	0.50	1.49	5.42	8.28	3.71	0.13
68	0.0100	3.62	2.47	2.96	1.20	4.25	0.50	1.49	5.35	8.08	3.64	0.12
68	0.0125	3.58	2.46	2.91	1.19	4.18	0.50	1.48	5.31	7.98	3.61	0.12
68	0.0150	3.55	2.45	2.87	1.17	4.12	0.50	1.48	5.28	7.88	3.58	0.12
68	0.0175	3.52	2.44	2.83	1.15	4.06	0.50	1.48	5.24	7.79	3.54	0.12
68	0.0200	3.49	2.44	2.79	1.13	4.00	0.50	1.48	5.21	7.69	3.51	0.12
68	0.0250	3.42	2.42	2.71	1.10	3.88	0.49	1.47	5.14	7.50	3.45	0.12
68	0.0300	3.37	2.41	2.63	1.07	3.77	0.49	1.46	5.07	7.31	3.39	0.11
68	0.0350	3.31	2.39	2.56	1.03	3.67	0.49	1.46	5.01	7.13	3.33	0.11
68	0.0400	3.25	2.38	2.49	1.00	3.57	0.49	1.45	4.91	6.96	3.28	0.11
69	0.0000	3.64	2.50	3.07	1.25	4.33	0.50	1.50	5.50	8.50	3.60	0.12
69	0.0050	3.57	2.48	2.98	1.21	4.20	0.50	1.49	5.42	8.14	3.54	0.12
69	0.0100	3.51	2.47	2.89	1.18	4.07	0.50	1.49	5.34	7.81	3.47	0.12
69	0.0125	3.48	2.46	2.85	1.16	4.01	0.50	1.48	5.30	7.65	3.44	0.11
69	0.0150	3.45	2.45	2.81	1.14	3.96	0.50	1.48	5.27	7.49	3.41	0.11
69	0.0175	3.42	2.44	2.77	1.13	3.90	0.50	1.48	5.23	7.34	3.38	0.11
69	0.0200	3.39	2.44	2.73	1.11	3.84	0.50	1.48	5.19	7.19	3.36	0.11
69	0.0250	3.33	2.42	2.66	1.08	3.74	0.49	1.47	5.10	6.91	3.30	0.11
69	0.0300	3.28	2.41	2.59	1.05	3.64	0.49	1.46	5.00	6.72	3.25	0.11
69	0.0350	3.22	2.39	2.52	1.02	3.54	0.49	1.46	4.90	6.61	3.19	0.10
69	0.0400	3.17	2.38	2.46	0.99	3.45	0.49	1.45	4.82	6.49	3.14	0.10
70	0.0000	3.47	2.50	2.96	1.28	4.44	0.50	1.50	4.50	7.50	3.48	0.15
70	0.0050	3.41	2.48	2.87	1.24	4.30	0.50	1.49	4.45	7.36	3.42	0.15
70	0.0100	3.35	2.47	2.79	1.21	4.17	0.50	1.49	4.40	7.23	3.36	0.14
70	0.0125	3.32	2.46	2.76	1.19	4.11	0.50	1.48	4.38	7.16	3.33	0.14
70	0.0150	3.29	2.45	2.72	1.17	4.05	0.50	1.48	4.35	7.09	3.31	0.14
70	0.0175	3.27	2.44	2.68	1.16	3.99	0.50	1.48	4.33	7.03	3.28	0.14
70	0.0200	3.24	2.44	2.65	1.14	3.94	0.50	1.48	4.30	6.97	3.25	0.14
70	0.0250	3.19	2.42	2.58	1.11	3.83	0.49	1.47	4.26	6.84	3.20	0.14
70	0.0300	3.14	2.41	2.51	1.08	3.72	0.49	1.46	4.21	6.72	3.15	0.13
70	0.0350	3.09	2.39	2.45	1.05	3.63	0.49	1.46	4.17	6.61	3.10	0.13
70	0.0400	3.04	2.38	2.39	1.02	3.53	0.49	1.45	4.12	6.49	3.06	0.13
71	0.0000	3.33	2.50	2.84	1.20	4.10	0.50	1.50	4.50	7.50	3.36	0.16
71	0.0050	3.27	2.48	2.77	1.17	3.99	0.50	1.49	4.45	7.36	3.30	0.16
71	0.0100	3.22	2.47	2.69	1.14	3.88	0.50	1.48	4.40	7.23	3.25	0.15
71	0.0125	3.20	2.46	2.66	1.12	3.83	0.50	1.47	4.38	7.16	3.22	0.15
71	0.0150	3.17	2.45	2.63	1.11	3.78	0.50	1.47	4.35	7.09	3.19	0.15

(continued)

Table 1
Characteristics of Present Value Probability Mass Functions for Initially Active Male High School Graduates (continued)

											Bootstrap Estimates	
Age	**NDR**	**Mean**	**Median**	**SD**	**SK**	**KU**	**10th**	**25th**	**75th**	**90th**	**Mean**	**SD**
71	0.0175	3.15	2.44	2.59	1.10	3.73	0.50	1.46	4.33	7.03	3.17	0.15
71	0.0200	3.12	2.44	2.56	1.08	3.68	0.50	1.46	4.30	6.97	3.15	0.15
71	0.0250	3.07	2.42	2.50	1.05	3.59	0.49	1.45	4.26	6.84	3.10	0.14
71	0.0300	3.03	2.41	2.44	1.03	3.50	0.49	1.44	4.21	6.72	3.05	0.14
71	0.0350	2.98	2.39	2.38	1.00	3.41	0.49	1.43	4.17	6.61	3.01	0.14
71	0.0400	2.94	2.38	2.33	0.97	3.33	0.49	1.42	4.12	6.49	2.96	0.14
72	0.0000	3.29	2.50	2.73	1.16	4.02	0.50	1.50	4.50	7.50	3.26	0.16
72	0.0050	3.24	2.48	2.66	1.13	3.91	0.50	1.49	4.45	7.36	3.21	0.16
72	0.0100	3.19	2.47	2.60	1.10	3.80	0.50	1.49	4.40	7.23	3.16	0.16
72	0.0125	3.17	2.46	2.56	1.08	3.75	0.50	1.48	4.38	7.16	3.14	0.15
72	0.0150	3.14	2.45	2.53	1.07	3.70	0.50	1.48	4.35	7.09	3.12	0.15
72	0.0175	3.12	2.44	2.50	1.05	3.66	0.50	1.48	4.33	7.03	3.09	0.15
72	0.0200	3.10	2.44	2.47	1.04	3.61	0.50	1.48	4.30	6.97	3.07	0.15
72	0.0250	3.05	2.42	2.42	1.01	3.52	0.49	1.47	4.26	6.84	3.03	0.15
72	0.0300	3.01	2.41	2.36	0.98	3.44	0.49	1.46	4.21	6.72	2.98	0.14
72	0.0350	2.97	2.39	2.31	0.96	3.35	0.49	1.46	4.17	6.61	2.94	0.14
72	0.0400	2.92	2.38	2.26	0.93	3.28	0.49	1.45	4.12	6.49	2.90	0.14
73	0.0000	3.17	2.50	2.62	1.13	3.88	0.50	1.50	4.50	7.50	3.14	0.16
73	0.0050	3.13	2.48	2.55	1.11	3.78	0.50	1.49	4.45	7.29	3.10	0.16
73	0.0100	3.08	2.47	2.49	1.08	3.69	0.50	1.49	4.40	7.08	3.05	0.16
73	0.0125	3.06	2.46	2.46	1.06	3.64	0.50	1.48	4.38	6.99	3.03	0.15
73	0.0150	3.04	2.45	2.43	1.05	3.60	0.50	1.48	4.35	6.89	3.01	0.15
73	0.0175	3.02	2.44	2.41	1.04	3.56	0.50	1.48	4.33	6.80	2.99	0.15
73	0.0200	3.00	2.44	2.38	1.02	3.51	0.50	1.48	4.30	6.70	2.97	0.15
73	0.0250	2.95	2.42	2.33	1.00	3.43	0.49	1.47	4.26	6.52	2.93	0.15
73	0.0300	2.91	2.41	2.27	0.97	3.36	0.49	1.46	4.21	6.35	2.89	0.14
73	0.0350	2.88	2.39	2.23	0.95	3.29	0.49	1.46	4.17	6.19	2.85	0.14
73	0.0400	2.84	2.38	2.18	0.92	3.22	0.49	1.45	4.12	6.03	2.81	0.14
74	0.0000	3.02	2.50	2.48	1.12	3.82	0.50	1.50	4.50	6.50	3.02	0.18
74	0.0050	2.97	2.48	2.43	1.10	3.73	0.50	1.47	4.45	6.40	2.98	0.18
74	0.0100	2.93	2.47	2.37	1.07	3.64	0.50	1.44	4.40	6.29	2.94	0.18
74	0.0125	2.91	2.46	2.35	1.06	3.60	0.50	1.43	4.38	6.24	2.92	0.17
74	0.0150	2.89	2.45	2.32	1.05	3.56	0.50	1.41	4.35	6.19	2.90	0.17
74	0.0175	2.88	2.44	2.29	1.03	3.52	0.50	1.40	4.33	6.14	2.88	0.17
74	0.0200	2.86	2.44	2.27	1.02	3.48	0.50	1.38	4.30	6.10	2.86	0.17
74	0.0250	2.82	2.42	2.22	1.00	3.40	0.49	1.36	4.26	6.00	2.83	0.17
74	0.0300	2.78	2.41	2.18	0.97	3.33	0.49	1.33	4.21	5.91	2.79	0.16
74	0.0350	2.75	2.39	2.13	0.95	3.26	0.49	1.31	4.17	5.82	2.76	0.16
74	0.0400	2.71	2.38	2.09	0.93	3.20	0.49	1.28	4.12	5.73	2.72	0.16
75	0.0000	2.86	2.50	2.34	1.08	3.70	0.50	0.50	4.50	6.50	2.85	0.17
75	0.0050	2.83	2.48	2.29	1.06	3.61	0.50	0.50	4.45	6.40	2.82	0.17
75	0.0100	2.79	2.47	2.24	1.04	3.53	0.50	0.50	4.40	6.29	2.78	0.16
75	0.0125	2.77	2.46	2.22	1.02	3.49	0.50	0.50	4.38	6.24	2.76	0.16
75	0.0150	2.76	2.45	2.19	1.01	3.45	0.50	0.50	4.35	6.19	2.74	0.16
75	0.0175	2.74	2.44	2.17	1.00	3.42	0.50	0.50	4.33	6.14	2.73	0.16
75	0.0200	2.72	2.44	2.15	0.99	3.38	0.50	0.50	4.30	6.10	2.71	0.16
75	0.0250	2.69	2.42	2.11	0.97	3.31	0.49	0.49	4.26	6.00	2.68	0.16
75	0.0300	2.66	2.41	2.06	0.95	3.24	0.49	0.49	4.21	5.91	2.64	0.15
75	0.0350	2.62	2.39	2.02	0.92	3.18	0.49	0.49	4.17	5.82	2.61	0.15
75	0.0400	2.59	2.38	1.99	0.90	3.12	0.49	0.49	4.12	5.73	2.58	0.15

Figure 3a. PMF for PV with NDR = .00 for Initially Active Male High School Graduates Age 30

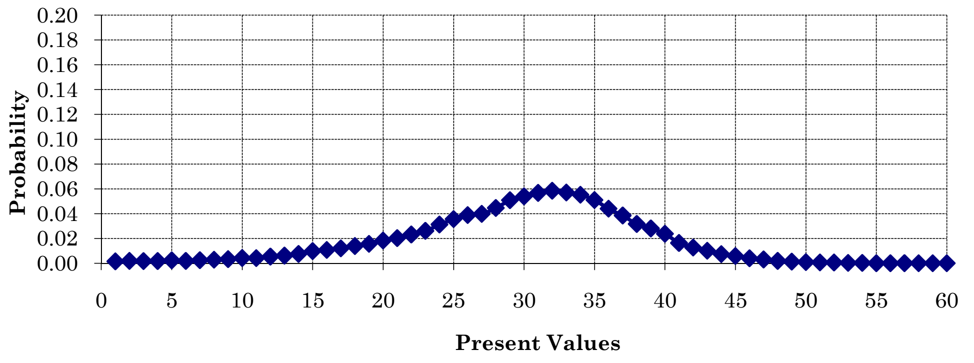

Figure 3b. PMF for PV with NDR = .02 for Initially Active Male High School Graduates Age 30

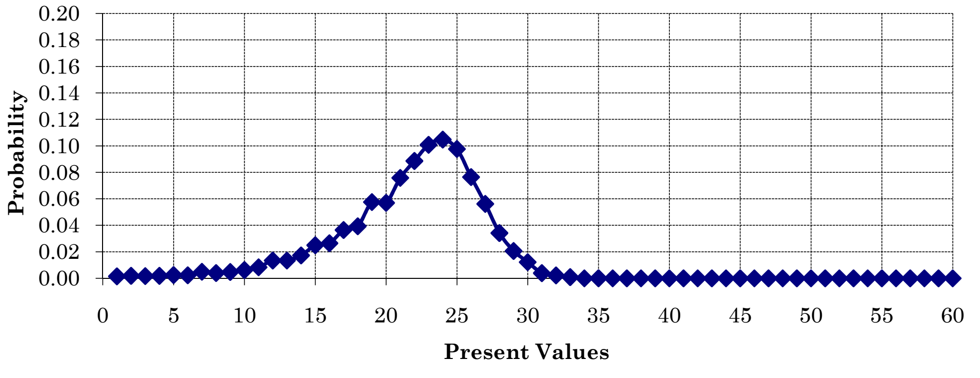

Figure 3c. PMF for PV with NDR = .04 for Initially Active Male High School Graduates Age 30

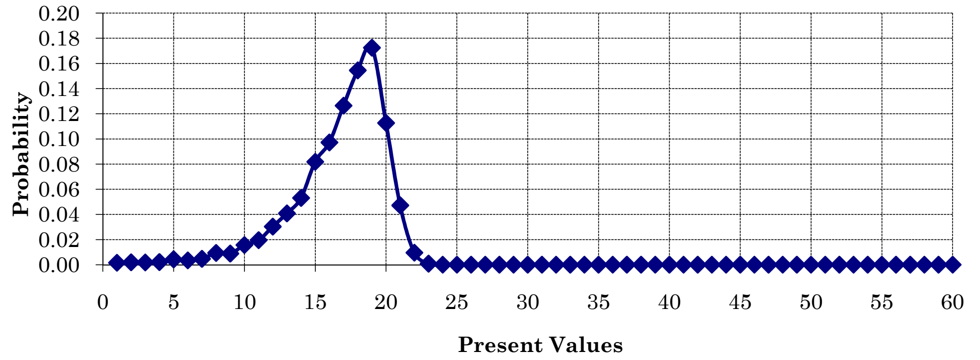

Table 2
Simulation Percentage Errors*

Net Discount Rates

Age	0.0000	0.0050	0.0100	0.0125	0.0150	0.0175	0.0200	0.0250	0.0300	0.0350	0.0400
16	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
18	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
19	0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
23	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
24	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
26	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
27	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
28	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
29	0.000	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
30	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
31	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
32	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
33	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	-0.001	-0.001
34	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
35	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001
36	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37	-0.002	-0.002	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
38	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
39	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
40	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
41	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
42	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000
43	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
44	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.003	-0.003	-0.003
45	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
46	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001
47	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
48	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
49	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
51	-0.004	-0.004	-0.004	-0.004	-0.004	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
52	-0.005	-0.005	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
53	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001	-0.001
54	-0.006	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005
55	0.002	0.002	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
56	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
57	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
58	-0.003	-0.003	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
59	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	0.000
60	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
61	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
62	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.005	-0.005	-0.005
63	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005	-0.006	-0.006	-0.006	-0.006
64	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001
65	-0.005	-0.005	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
66	-0.002	-0.002	-0.002	-0.002	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
67	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
68	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002
69	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
70	-0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
71	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.004	0.004	0.004	0.004
72	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
73	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006
74	-0.011	-0.011	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010
75	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002

*Expected present values from simulation less expected present values from expectancy theorem divided by expected present values from expectancy theorem.

VII. Relation to the Ogden Tables

In England and Wales, there has been, and remains, a desire not to use experts in determining economic damages in tort cases. When experts were used, they have tended to be actuaries. The approach before 1984 had been to allow judges to make damages determinations based on whatever method they may have chosen, including inaccurate, unscientific intuition. This rejection of economic, actuarial, and statistical evidence came under increasing criticism, and led to the Ogden Tables. Their first edition dates to 1984, and they are named after Sir Michael Ogden, who headed the first Working Party (an inter-disciplinary group of actuaries, lawyers, accountants and other interested parties). The charge was to devise "the simple calculation of full compensation for those victims who have suffered future loss as a result of wrongful injury" (Government Actuary's Office, *Actuarial Tables*, 2004).

The relevant Ogden Tables are Numbered 3 through 14 (providing base-line multipliers), and Supplementary Tables A-D (providing reduction factors). The annual amount of earnings lost is known as a multiplicand, while the product of the base-line multiplier and reduction factor is known as the multiplier. Damages result from multiplying the multiplier and the multiplicand. In England and Wales since 1999, after the appearance of the Third Edition, judges (who determine economic damages in tort cases) have been required to consult present values using the Ogden Tables, which are similar in some ways to those presented here. Comparisons between the calculations done in the American and U.K. systems were the subject of papers and discussion in the 2004 International Conference of the National Association of Forensic Economics in Edinburgh, Scotland. The May, 2005 International Conference in Dublin continued the theme. (For additional background on the system in England and Wales, see Haberman and Bloomfield, 1990; Lewis, et al., 2003; Butt, et al., 2006; Butt et al., 2008; Ciecka, 2008; Skoog, 2008; and Skoog and Ciecka, 2009.)

The tables are in their Sixth Edition (2007), which contains a major change in its Section B Supplementary Tables A-D. Their purpose, from paragraph 2 of the Introduction, seems less ambitious as compared to the Fifth Edition—they are "designed to help in the calculation of future pecuniary losses."

The base multipliers are (continuous) annuities certain, written from an age, which varies in each table, to one of the terminal retirement ages of 50, 55, 60, 65, 70 or 75. There are different tables for men and women. The reduction factors incorporate age, sex, whether the initial state is employed or not employed, educational level, age and whether the person is disabled or not. They are meant to incorporate considerations for not working other than mortality. Earlier tables had incorporated considerations such as occupation, industrial sector, geographic location, and the level of economic activity, but these are now abandoned. The approach is broad brush, since several ages are grouped together into the same reduction factor, all reduction factors are calculated only at an *NDR* of 2 ½% (the current legal requirement), and there is an attempt to control for disability. The Markov model is between employment and its negation ("not employed") as opposed to the U.S. worklife notion of labor force participation.

VIII. Thoughts on the Theoretical Place of Expected Present Values and Present Value Probability Mass Functions

We conclude with a brief discussion of the possible uses of expected present values and pmf's of present value random variables. First, any place a worklife table $(\text{with } NDR = 0)$ is currently published, an expanded table with *NDR* varying might also be published. Exploiting the *Expected Present Value Theorem* will result in additional information at less computer cost and serve as an accuracy check for simulations designed to capture entire probability mass functions. Tables similar to Table 1 would provide standard deviations and other distributional characteristics (*e.g.*, medians) of present values as well as standard errors of the sample mean present value. Second, since interest is ultimately on present values, and if the analyst currently allocates worklife on some *ad hoc* basis, or uses Skoog-Ciecka (2006) to correct for front or uniform loading, tables with the new present value function would correctly perform the allocation of worklife and at the same time provide the desired object. Third, in conjunction with these two reasons, some forensic economists do not opine on the base period rate of loss: in essence, their opinion is about one of the function values in a present value table. Those opinions may allocate worklife expectancy on an *ad hoc* basis. Present value tables permit not only a check on those calculations but provide a measure of the *ad hoc* error. Fourth, adding worklife (active) and leisure-life (inactive) values together permits the valuation of a flow over all years out to truncation age; and the sum of the expected values of active and inactive present value random variables equals the expected present value of a life annuity. This sum would properly allocate life care services over the years to *TA* assuming a normal survival function.

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