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The Markov (Increment-Decrement) Model of Labor Force Activity: New Results Beyond Work-Life Expectancies

Introduction

The Bureau of Labor Statistics introduced the Markov (increment-decrement) model of labor force activity in *Bulletin* 2135 in 1982. A subsequent BLS publication, *Bulletin* 2254, in 1986 also used the increment-decrement methodology.[1] Arguably, work-life expectancies have been the most important progeny of this methodology; but the purpose of this paper is to further explore the rich implications of the Markov nature of the increment-decrement model – implications that take us far beyond the concept of work-life expectancy. We show that the increment-decrement model is a valuable construct that enables us to describe many aspects of labor market activity that should be of interest to economists, sociologists, and demographers.

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The Probability Mass Function for Years of Activity

Our point of departure is a variant of an illustration in *Bulletin* 2135 that depicts alternative paths of survival and labor force attachment for those who are initially inactive in the labor force. Figure 1 below is a similar illustration, but it is for initially active individuals. It begins with individuals who are active at exact age x, and then it traces all possible paths to age $(x+4)$. Once transition probabilities have been estimated, we can think of figure 1 as a road map to calculating the probability of an initially active person spending various numbers of years in active and inactive states. We use the usual notation: let ${}^{j}p_x^{k}$ denote the transition probability that a person in state j at exact age x will be in state k at age $(x+1)$, where $j \in (a,i)$, $k \in (a,i,d)$, and a, i, and d denote the active, inactive, and death states, respectively. In addition, let Y_x denote an additional-years-of-activity random variable with $p(x,a,y)$ being the probability that an active individual at age x will accumulate $Y_x = y$ years of activity from age x to the end of his or her lifetime. We call $p(x,a,y)$ the probability mass function (pmf) for years of activity for initially active individuals.[2]

Assume that all labor market activity ends at truncation age $TA = 80$, and the last year for which we have transition probabilities is age 79. Then the pmf for a person age 79 is simply

(1)

$$p(79,a,y) = \begin{cases} {}^{a}p_{79}^{a} & \text{for y = 1} \\ {}^{a}p_{79}^{i} + {}^{a}p_{79}^{d} & \text{for y = 0,} \end{cases}$$

assuming that state transitions occur at the beginning of the age interval from 79 to 80.[3] In figure 1, assume that age $x = 79$ and that the triangle of paths ends at age $x = 80$, *i.e.*, it only has two rows. The probability of one year of activity is simply the probability of taking the a to a path which is ${}^{a}p_{79}^{a}$, and the probability of zero years of activity is the sum of the probabilities of taking the a to i path and the a to d path which is ${}^{a}p_{79}^{i} + {}^{a}p_{79}^{d}$.

The pmf for an initially active 78-year-old person would be computed in the same manner with the modification that the triangle of paths would have three rows which start at $x = 78$, with an additional row for age $(x+1) = 79$, and a final row for $(x+2) = 80$.

The pmf becomes

(2)

$$p(78,a,y)=\begin{cases} {}^a p_{78}^a \, {}^a p_{79}^a & \text{for } y=2 \\ {}^a p_{78}^a \, {}^a p_{79}^d + {}^a p_{78}^a \, {}^a p_{79}^i + {}^a p_{78}^i \, {}^i p_{79}^a & \text{for } y=1 \\ {}^a p_{78}^d + {}^a p_{78}^i \, {}^i p_{79}^i + {}^a p_{78}^i \, {}^i p_{79}^d & \text{for } y=0 \end{cases}$$

if we once again assume beginning of period transitions.

In principal, the pmf for any age could be computed in the same manner as (1) and (2). However, the number of paths and their length become unwieldy as we consider younger ages. We can easily get a lower bound estimate of the number of possible paths at any age by simply excluding the death state which, if it occurs, terminates a path as illustrated in figure 1. For example, there are $2^{79-16+1}=1.845\times10^{19}$ such paths if labor force activity commences at a beginning age $BA=16$, requiring almost 600,000 years of computer time if a single computer were to trace a million paths per second. Such a staggering number of calculations naturally leads us to search for more efficient methods to find the pmf, but the final result would have to be equivalent to having searched all the paths in an appropriately expanded version of figure 1.

Probability mass functions are defined, and efficiently computed, by a set of global conditions (that hold regardless of whether transitions occur at the beginning, mid-point, or end of a period), boundary conditions (that hold for 0, .5, and 1 year of activity, depending on when transitions occur within a period), and main recursions (that are interior to the ages treated by the boundary conditions). These conditions are stated below and are taken from Skoog (2002) and Skoog and Ciecka (2002).

Global Conditions:

(3a) $p(x,a,r)=p(x,i,r)=0$ if $r<0$ or $r>TA-x$

(3b) $p(TA,a,0)=p(TA,i,0)=1$

(3c) ${}^a p_x^a = {}^i p_x^a = 0$ for $x \ge TA$

Condition (3a) merely expresses the idea that there is a zero probability of accumulating negative years of activity or years of activity in excess of the number of years to the truncation age whether starting age x in the active or inactive state. (3b) states that the probability is one that a person age $x = TA$ accumulates zero years of activity. Since (3c) says that labor force activity ceases at age $x = TA$, it also implies (3b) and illustrates the idea that we seek boundary and global conditions on the mass functions with, or implied by, conditions on the underlying transition probabilities.

Boundary Conditions:

(4a) $$p(x,a,0) = {}^{a}p_x^{i}\,p(x+1,i,0) + {}^{a}p_x^{d} \quad x = BA,\ldots,TA-1$$

(4b) $$p(x,i,y) = {}^{i}p_x^{i}\,p(x+1,i,0) + {}^{i}p_x^{d} \quad x = BA,\ldots,TA-1$$

Condition (4a) expresses the observation that there are only two ways a person active at age x can experience no additional years of activity: die or turn inactive and repeat the process. In (4b) a person inactive at age x accumulates no years of future activity by remaining inactive or through death. The remainder of the probability mass values are defined by the main recursions in (5a) and (5b).

Main Recursions:

(5a) $$p(x,a,y) = {}^{a}p_x^{a}\,p(x+1,a,y-1) + {}^{a}p_x^{i}\,(x+1,i,y)$$

(5b) $$p(x,i,y) = {}^{i}p_x^{a}\,p(x+1,a,y-1) + {}^{i}p_x^{i}\,p(x+1,i,y)$$

In (5a) and (5b), $x = BA,\ldots,TA-1$ and $y = 1,2,\ldots,TA-x$. The right-hand side of (5a) is the sum of two terms that contribute to the probability that an active person age x will accumulate y years of activity; $p(x+1,a,y-1)$ and $p(x+1,i,y)$ are the probabilities of experiencing $y-1$ active years when being active at age $x+1$ and the probabilities of experiencing y active years when being inactive at age $x+1$, respectively; the probability of y years of activity results when the former probability is multiplied by ${}^{a}p_x^{a}$ (thereby yielding one more year of activity) and the latter multiplied by ${}^{a}p_x^{i}$ (causing activity to remain at y years). A similar interpretation holds in (5b) where ${}^{i}p_x^{a}$ leads to one additional year of activity but ${}^{i}p_x^{i}$ does not increase years of activity.

Characteristics of the Probability Mass Function for Years of Activity

The probability mass function for Y_x can be used to define the expected value of additional years of activity (*i.e.*, work-life expectancy[4]) for actives at age x as[5]

(6a) $$E(Y_x) = WLE_x(a) = \sum_{y=0}^{80-x} yp(x,a,y) = {}^a e_x^a .$$

However, work-life expectancy is only one of the many statistics that can be computed once the pmf is in hand; knowledge of the pmf also enables us to specify other properties of labor market activity that were heretofore impossible to calculate. That is, the pmf allows us to explore all of the implications of the increment-decrement model that are not as transparent as work-life expectancy but are useful to understanding labor market activity. For example, the variance $V(Y_x)$, standard deviation $SD(Y_x)$, median $med(Y_x)$, mode $mode(Y_x)$, skewness $s(Y_x)$, and kurtosis $k(Y_x)$ can now be calculated in the usual manner as specified in (6b)–(6g).

(6b) $$V(Y_x) = \sum_{y=0}^{80-x} \left[y - WLE_x(a) \right]^2 p(x,a,y)$$

(6c) $$SD(Y_x) = \sqrt{V(Y_x)}$$

(6d) $$\sum_{y=0}^{med(Y_x)} p(x,a,y) = .5 \text{ , or interpolation between } m \text{ and } m+1$$

$$\text{such that } \sum_{y=0}^{m} p(x,a,y) < .5 \text{ and } \sum_{y=0}^{m+1} p(x,a,y) > .5$$

(6e) $$mode(Y_x) = \max_y \left[p(x,a,y) \right]$$

$$(6f) \qquad s(Y_x) = \frac{\sum_{y=0}^{80-x} \left[y - WLE_x(a) \right]^3 p(x,a,y)}{V(Y_x)^{1.5}}$$

$$(6g) \qquad k(Y_x) = \frac{\sum_{y=0}^{80-x} \left[y - WLE_x(a) \right]^4 p(x,a,y)}{\left[V(Y_x) \right]^2}$$

All of the statistics in (6b)-(6g) are new results describing labor market activity; and (6a) is an entirely new way to look at the familiar concept of work-life expectancy, *i.e.*, as the mean value of a random variable.

Some Empirical Results

Table 1 shows the values of the foregoing parameters for all active men and women at ages $x = 20, 35, 50, 65$ using transition probabilities computed from the *Current Population Survey* for 1997 and 1998 by Ciecka, Donley, and Goldman (2000).[6] The pmf for both men and women are negatively skewed at young ages and then become positively skewed, but the transformation from negative to positive skewness occurs at younger ages for women than men (see table 1 and figures 2 and 3). For men ages 20 to approximately 47, the pmf has negative skewness, with mean (*i.e.*, work-life expectancy) smaller than the median which is smaller than the mode. As age increases to 47, skewness declines (in absolute value), and the pmf for men become more symmetric; but this pattern reverses from ages 47 to 65 when skewness becomes positive and increases with age. At approximately age 47, the pmf is not only symmetric but it is approximately normal. (Its skewness and kurtosis are -.05 and 2.86 whereas a normal variate has zero skewness and kurtosis of 3.0, respectively.) Variances and standard deviations decline with age, reflecting the smaller range of possible years of activity as age increases. However, coefficients of variation (i.e., the standard deviation divided by the mean) increase with age for men. For women ages 20-43 the pmf has negative skewness and is positively skewed thereafter to age 65 (see figure 3). The pmf is approximately normal at age 43 when its skewness is -.03 and kurtosis is 2.71. As with men, variances and standard deviations decline with age, and the coefficients of variation increase with age. At any given age in table 1, the mean, median, mode, and variance for men exceeds that of women (except at age 65 when both have a zero mode); but the coefficient of variation is slightly smaller for men.

Figure 1. Triangle of paths

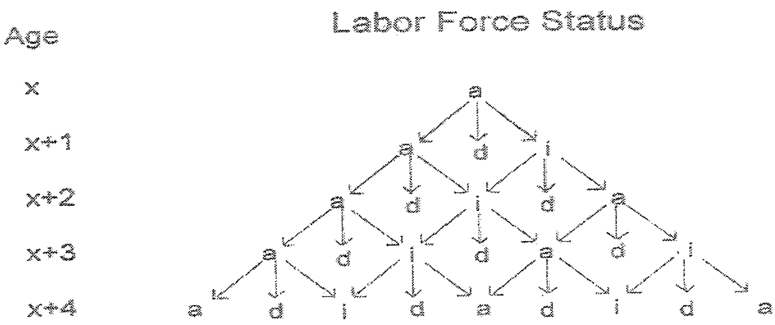


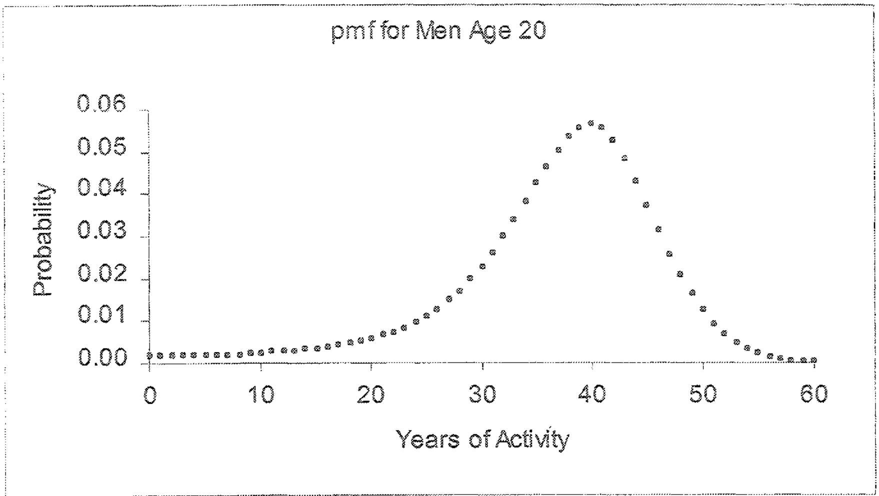
Table 1. Properties of Years-of-Activity Random Variable at Various Ages for Initially Active Men and Women

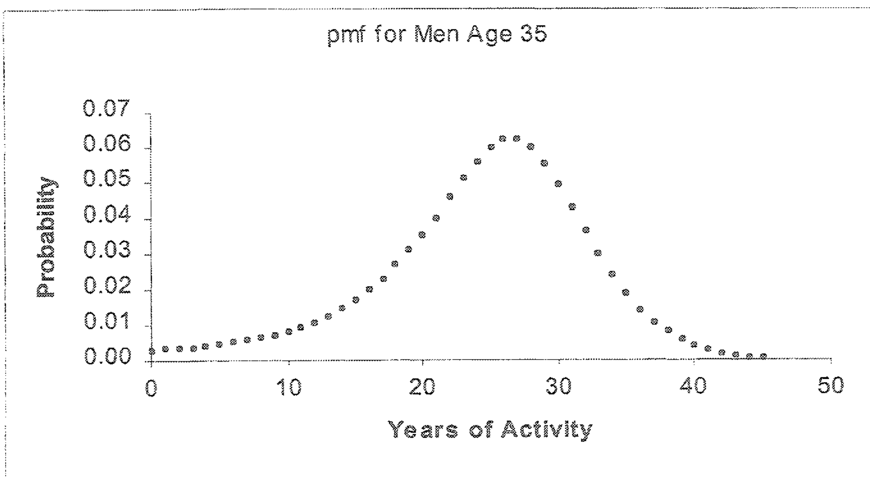
Property	Age 20	Age 35	Age 50	Age 65
Men				
Mean (WLE)	36.74	24.48	12.08	3.65
Median	37.74	24.87	11.55	2.43
Mode	40.00	26.00	12.00	0.00
Variance	87.79	57.31	30.30	10.22
Standard Deviation	9.37	7.57	5.50	3.20
Coefficient of Variation	.26	.31	.46	.88
Skewness	-1.14	-.62	.13	1.02
Kurtosis	5.00	3.57	2.82	3.67
Women				
Mean (WLE)	31.51	20.98	10.08	3.08

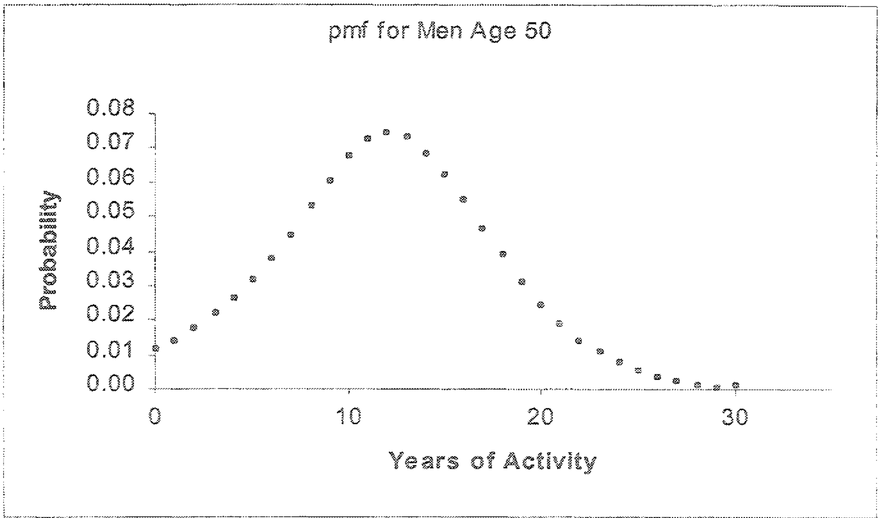
Property	Age 20	Age 35	Age 50	Age 65
Women, continued				
Median	31.60	20.88	9.42	1.86
Mode	33.00	23.00	10.00	0.00
Variance	72.84	52.91	28.36	8.34
Standard Deviation	8.53	7.27	5.33	2.89
Coefficient of Variation	.27	.35	.53	.94
Skewness	-.42	-.21	.30	1.14
Kurtosis	3.33	2.88	2.75	4.03

Tables 2–5 contain the complete pmf for initially active men at ages $x = 20, 35, 50, 65$. The counterpart pmf for women are in tables 6–9. We consider a 20-year-old male in order to exemplify some uses of these tables in conjunction with table 1. The work-life expectancy for such a person is 36.74 years as shown in table 1. Although based on 1997-98 data, this work-life expectancy is not very different from the expectancy of 37.4 years reported in *Bulletin* 2254 which is based on 1979-80 labor force activity. In fact, these two expectancies are even closer to each other because the work-life of 36.74 years reported in this paper is based on beginning of period transitions. It would be approximately, .5 years larger, or 37.24 years, if based on mid-point transitions as used in *Bulletin* 2254. The latter figure, 37.24 years, is approximately equal to the work-life expectancy of 37.29 years reported by Ciecka, Donley, and Goldman (2000) even though they calculate work-life expectancies in the traditional manner developed by the BLS. At this point comparisons with previously published work cease because earlier work only focused on work-life expectancy. Our table 1 shows new summary statistics that take us far beyond work-life expectancy; and table 2 exhibits the entire pmf for 20-year-old men, which is graphed in figure 2. We now know all of the moments of the years-of-activity random variable including the variance, skewness and kurtosis (as described above). In addition to the summary statistics, one might be interested in calculating various probability intervals and the probabilities associated with specific events. For example, the smallest interval that contains 50% of the probability of years of activity might be of interest.[7] This interval is approximately 35 years to 44 years of activity, which we write as (35 Years, 44 Years) or simply as (35,44); using Table 2, the reader can verify that the sum of probabilities over this interval is 50.46% and is smaller than any other interval containing probability of at least 50%. So we can say that the probability is slightly over 50% that an

Figure 2. Probability Mass Functions for Initially Active Men at Various Ages






pmf for Men Age 50
Probability
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0
10
20
30
Years of Activity

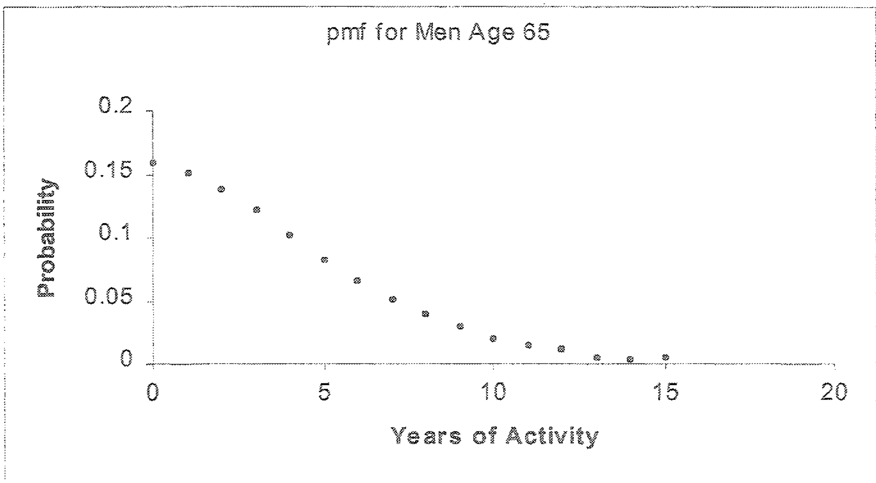
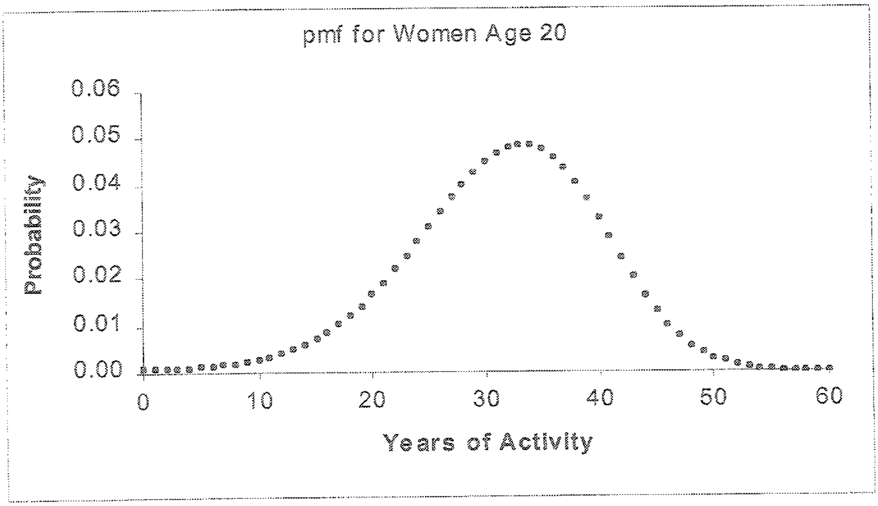
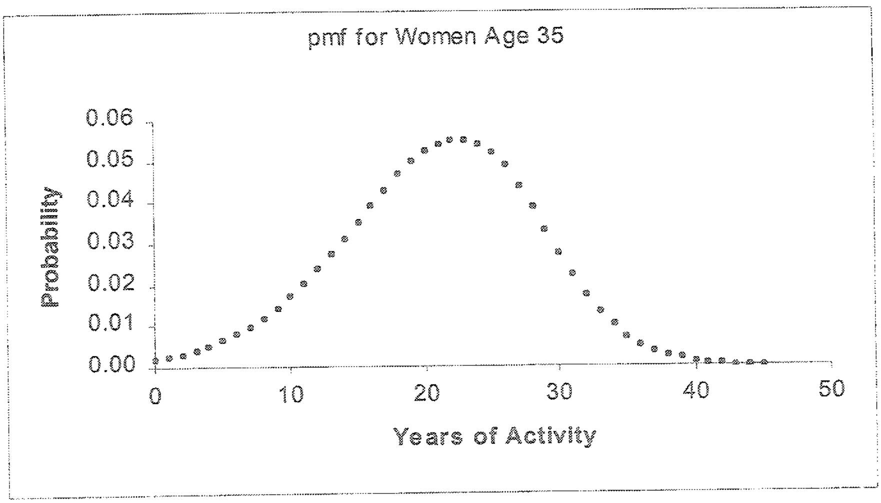
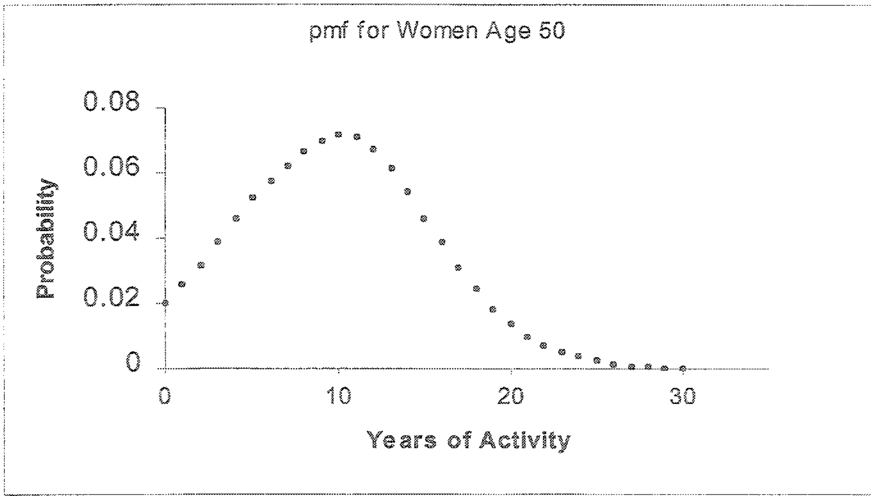
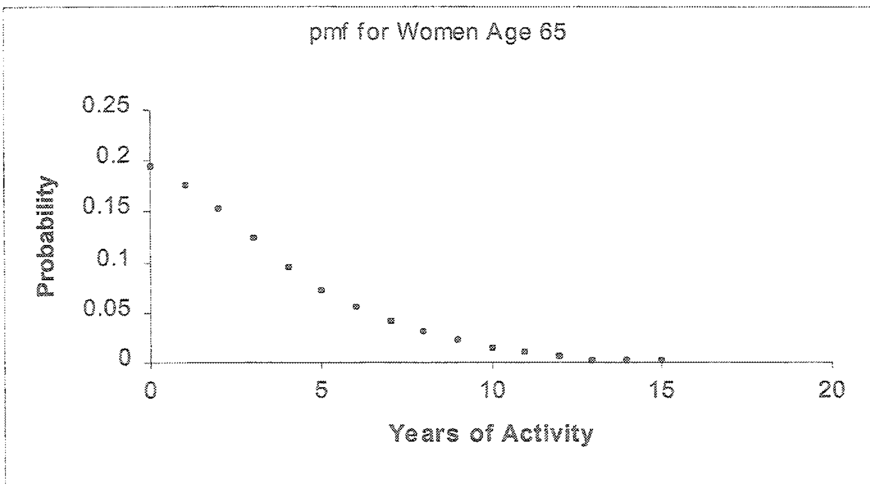

pmf for Men Age 65
Probability
0.2
0.15
0.1
0.05
0
0
5
10
15
20
Years of Activity

Figure 3. Probability Mass Functions for Initially Active Women at Various Ages






pmf for Women Age 50
0.08
0.06
0.04
0.02
0
Probability
0
10
20
30
Years of Activity


pmf for Women Age 65
0.25
0.2
0.15
0.1
0.05
0
Probability
0
5
10
15
20
Years of Activity

active 20-year-old male will be in the labor force between 35 and 44 years.[8] Notice that this interval includes the mean (work-life) of 36.74 years of activity; but the mean is closer to the left than the right end point of the interval, suggesting a greater probability that years of activity will exceed the mean rather than be smaller than the mean.[9]

Other probability intervals of interest might be the inter-quartile range and a 10%–90% interval. The inter-quartile range is (32, 42), *i.e.*, when excluding 25% of the probability in both tails of the pmf, there is a 50% probability of being in the labor force between 32 and 42 years. The 10%–90% probability interval is (24, 46), meaning that the probability is 80% that labor force activity will be between 24 and 46 years when 10% of the probability is excluded in each tail of the pmf. Of course, any other probability interval can be specified and the corresponding years of activity calculated from table 2. Reversing this procedure yields the probability of an event stated in terms of years of activity or a range of years. For example, what is the probability of a 20-year-old active male accruing more than 45 years of activity in his future? From table 2, this probability is $.1374 = 1.0000 - .8626$. Another question of interest might be: What is the probability that years of activity will be within one standard deviation of work-life expectancy? The interval in question is $36.74 \pm 9.37 \approx (27, 46)$; and from table 2, we know this probability is $.7757 = .8940 - .1183$.

Table 2. Probability Mass Function for Years of Activity for Initially Active Men Age 20

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
0	0.0021	0.0021	31	0.0262	0.2188
1	0.0021	0.0042	32	0.0299	0.2487
2	0.0020	0.0062	33	0.0339	0.2827
3	0.0020	0.0082	34	0.0381	0.3208
4	0.0019	0.0101	35	0.0424	0.3631
5	0.0019	0.0120	36	0.0465	0.4096
6	0.0019	0.0139	37	0.0504	0.4601
7	0.0020	0.0159	38	0.0537	0.5137
8	0.0022	0.0181	39	0.0557	0.5695
9	0.0023	0.0204	40	0.0565	0.6260
10	0.0025	0.0229	41	0.0554	0.6814
11	0.0027	0.0256	42	0.0526	0.7339
12	0.0029	0.0284	43	0.0484	0.7823
13	0.0031	0.0315	44	0.0430	0.8253

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
14	0.0033	0.0348	45	0.0373	0.8626
15	0.0036	0.0384	46	0.0314	0.8940
16	0.0039	0.0422	47	0.0258	0.9198
17	0.0043	0.0465	48	0.0208	0.9406
18	0.0047	0.0512	49	0.0162	0.9568
19	0.0052	0.0565	50	0.0124	0.9692
20	0.0059	0.0623	51	0.0094	0.9786
21	0.0066	0.0689	52	0.0069	0.9855
22	0.0075	0.0764	53	0.0050	0.9905
23	0.0085	0.0848	54	0.0035	0.9940
24	0.0096	0.0945	55	0.0023	0.9963
25	0.0111	0.1055	56	0.0016	0.9979
26	0.0128	0.1183	57	0.0010	0.9989
27	0.0148	0.1331	58	0.0006	0.9994
28	0.0171	0.1502	59	0.0003	0.9997
29	0.0197	0.1699	60	0.0003	1.0000
30	0.0228	0.1927			

Table 3. Probability Mass Function for Years of Activity for Initially Active Men Age 35

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
0	0.0030	0.0030	23	0.0512	0.3918
1	0.0032	0.0063	24	0.0560	0.4478
2	0.0035	0.0098	25	0.0603	0.5081
3	0.0038	0.0135	26	0.0626	0.5706
4	0.0041	0.0177	27	0.0624	0.6330
5	0.0046	0.0223	28	0.0602	0.6932
6	0.0051	0.0274	29	0.0555	0.7486
7	0.0057	0.0331	30	0.0497	0.7983
8	0.0064	0.0395	31	0.0431	0.8414
9	0.0071	0.0466	32	0.0363	0.8777
10	0.0081	0.0547	33	0.0301	0.9078
11	0.0093	0.0640	34	0.0240	0.9318
12	0.0107	0.0747	35	0.0187	0.9505
13	0.0125	0.0872	36	0.0143	0.9648
14	0.0145	0.1016	37	0.0108	0.9756
15	0.0169	0.1185	38	0.0080	0.9836
16	0.0198	0.1383	39	0.0057	0.9893
17	0.0230	0.1613	40	0.0039	0.9932
18	0.0269	0.1882	41	0.0027	0.9959
19	0.0310	0.2192	42	0.0019	0.9978
20	0.0355	0.2547	43	0.0010	0.9987
21	0.0403	0.2950	44	0.0005	0.9993
22	0.0456	0.3406	45	0.0007	1.0000

Table 4. Probability Mass Function for Years of Activity for Initially Active Men Age 50

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
0	0.0114	0.0114	16	0.0547	0.7923
1	0.0143	0.0257	17	0.0465	0.8388
2	0.0176	0.0433	18	0.0390	0.8778
3	0.0219	0.0653	19	0.0313	0.9091
4	0.0265	0.0918	20	0.0245	0.9336
5	0.0318	0.1236	21	0.0189	0.9524
6	0.0378	0.1614	22	0.0143	0.9668
7	0.0449	0.2062	23	0.0108	0.9776
8	0.0529	0.2591	24	0.0077	0.9853
9	0.0603	0.3194	25	0.0053	0.9905
10	0.0675	0.3869	26	0.0037	0.9943
11	0.0725	0.4593	27	0.0026	0.9968
12	0.0742	0.5335	28	0.0013	0.9982
13	0.0732	0.6067	29	0.0007	0.9989
14	0.0686	0.6753	30	0.0011	1.0000
15	0.0623	0.7375			

Table 5. Probability Mass Function for Years of Activity for Initially Active Men Age 65

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
0	0.1585	0.1585	8	0.0401	0.9101
1	0.1516	0.3100	9	0.0294	0.9395
2	0.1373	0.4474	10	0.0205	0.9599
3	0.1220	0.5694	11	0.0151	0.9750
4	0.1018	0.6712	12	0.0110	0.9860
5	0.0821	0.7533	13	0.0055	0.9915
6	0.0654	0.8187	14	0.0029	0.9944
7	0.0513	0.8699	15	0.0056	1.0000

Table 6. Probability Mass Function for Years of Activity for Initially Active Women Age 20

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
0	0.0008	0.0008	31	0.0466	0.4711
1	0.0008	0.0016	32	0.0478	0.5190
2	0.0009	0.0025	33	0.0484	0.5674
3	0.0009	0.0034	34	0.0482	0.6156

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
4	0.0010	0.0044	35	0.0473	0.6629
5	0.0012	0.0056	36	0.0456	0.7086
6	0.0014	0.0070	37	0.0433	0.7519
7	0.0016	0.0086	38	0.0403	0.7922
8	0.0019	0.0105	39	0.0368	0.8290
9	0.0023	0.0128	40	0.0329	0.8618
10	0.0027	0.0155	41	0.0286	0.8905
11	0.0033	0.0188	42	0.0243	0.9148
12	0.0040	0.0228	43	0.0202	0.9349
13	0.0049	0.0277	44	0.0163	0.9513
14	0.0059	0.0336	45	0.0129	0.9642
15	0.0071	0.0407	46	0.0100	0.9742
16	0.0085	0.0492	47	0.0076	0.9817
17	0.0101	0.0593	48	0.0056	0.9873
18	0.0120	0.0713	49	0.0041	0.9914
19	0.0141	0.0854	50	0.0029	0.9943
20	0.0164	0.1019	51	0.0020	0.9963
21	0.0190	0.1208	52	0.0014	0.9977
22	0.0218	0.1426	53	0.0009	0.9986
23	0.0247	0.1673	54	0.0006	0.9992
24	0.0278	0.1951	55	0.0004	0.9996
25	0.0309	0.2260	56	0.0002	0.9998
26	0.0340	0.2600	57	0.0001	0.9999
27	0.0371	0.2971	58	0.0001	0.9999
28	0.0400	0.3371	59	0.0000	1.0000
29	0.0426	0.3797	60	0.0000	1.0000
30	0.0448	0.4245			

Table 7. Probability Mass Function for Years of Activity for Initially Active Women Age 35

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
0	0.0021	0.0021	23	0.0549	0.6161
1	0.0026	0.0048	24	0.0539	0.6700
2	0.0033	0.0081	25	0.0518	0.7218
3	0.0041	0.0122	26	0.0487	0.7704
4	0.0052	0.0173	27	0.0440	0.8144
5	0.0064	0.0238	28	0.0386	0.8530
6	0.0080	0.0317	29	0.0331	0.8861
7	0.0098	0.0415	30	0.0273	0.9134
8	0.0119	0.0534	31	0.0222	0.9355
9	0.0143	0.0678	32	0.0175	0.9531
10	0.0171	0.0849	33	0.0134	0.9664
11	0.0202	0.1051	34	0.0100	0.9764
12	0.0237	0.1287	35	0.0073	0.9837
13	0.0274	0.1561	36	0.0053	0.9890

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
14	0.0311	0.1872	37	0.0038	0.9928
15	0.0350	0.2222	38	0.0027	0.9955
16	0.0389	0.2611	39	0.0018	0.9973
17	0.0428	0.3039	40	0.0011	0.9984
18	0.0466	0.3504	41	0.0007	0.9992
19	0.0499	0.4003	42	0.0004	0.9996
20	0.0523	0.4526	43	0.0002	0.9998
21	0.0539	0.5065	44	0.0001	0.9999
22	0.0548	0.5613	45	0.0001	1.0000

Table 8. Probability Mass Function for Years of Activity for Initially Active Women Age 50

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
0	0.0199	0.0199	16	0.0385	0.8806
1	0.0256	0.0455	17	0.0313	0.9119
2	0.0317	0.0771	18	0.0243	0.9362
3	0.0388	0.1159	19	0.0183	0.9545
4	0.0460	0.1619	20	0.0135	0.9681
5	0.0522	0.2140	21	0.0100	0.9780
6	0.0575	0.2715	22	0.0073	0.9854
7	0.0623	0.3337	23	0.0053	0.9906
8	0.0668	0.4005	24	0.0036	0.9942
9	0.0697	0.4702	25	0.0023	0.9966
10	0.0714	0.5416	26	0.0016	0.9982
11	0.0713	0.6128	27	0.0010	0.9991
12	0.0673	0.6801	28	0.0004	0.9996
13	0.0615	0.7416	29	0.0002	0.9998
14	0.0545	0.7961	30	0.0002	1.0000
15	0.0460	0.8421			

Table 9. Probability Mass Function for Years of Activity for Initially Active Women Age 65

Years of Activity	Probability	Cumulative Probability	Years of Activity	Probability	Cumulative Probability
0	0.1938	0.1938	8	0.0311	0.9402
1	0.1759	0.3697	9	0.0222	0.9624
2	0.1522	0.5219	10	0.0145	0.9769
3	0.1230	0.6450	11	0.0104	0.9874
4	0.0955	0.7404	12	0.0066	0.9939
5	0.0721	0.8125	13	0.0030	0.9969
6	0.0548	0.8674	14	0.0013	0.9982
7	0.0417	0.9091	15	0.0018	1.0000

Conclusion

This paper places the concept of work-life expectancy into a broader perspective by showing that it is just one parameter of the additional-years-of-activity random variable. Since the pmf of future activity is now known, all of its parameters can be calculated. Tables 1–9 and figures 2 and 3 provide information that has heretofore been unknown, including the ability to make probability statements about years of activity. Questions involving error potential or probability associated with years of labor force activity estimates can now be addressed and answered. Recognizing that it is the Markov property of the increment-decrement model that gives rise to the pmf and all of the results described in this paper, the techniques that we develop here have been applied to other labor force topics such as years to final separation from the labor force (Skoog and Ciecka, 2003).

Table 1 provides various statistics for ages 20, 35, 50, and 65 for initially active men and women without regard to educational attainment. However, practical applications require much more finely age-calibrated tables as well as tables for educational attainment and for initial labor force status. Such tables appear in a separate paper in this issue of the *Journal of Legal Economics* (Skoog and Ciecka, 2001). There are 24 tables in all, consisting of separate tables for men and women, tables for those initially active and initially inactive, and tables for five education levels plus a table for all levels of education combined. Each table contains measures of central tendency, shape, and probability intervals.

Endnotes

1. The Markov model is defined and explained in Bureau of Labor Statistics *Bulletin* 2135 (1982), *Bulletin* 2254 (1986), Ciecka, Donley, and Goldman (2000), and Skoog (2002). In the Markov model, a person's transition probability for movement from state to state (here, active or inactive in the labor force) depends only on that person's current state.

2. Transition probabilities are primitives in this paper which we assume are known with certainty and which enable us to define the pmf for years of activity. All the statistics we compute arise from the inherent variation in years of activity and are not due to sampling error in estimating transition probabilities. The inclusion of transition probability sampling error would only slightly increase the dispersion in the years-of-activity random variable.

3. Beginning-of-period transitions are assumed throughout this paper. End-of-period transitions also could have been used; and, perhaps most realistically, mid-period transitions could be utilized (see Skoog and Ciecka, 2002). The pmf will change as the point of transition changes. Relative to beginning of period transitions, the pmf shifts approximately .5 years and 1.0 year to the right with mid-period and end of period transitions. The central ideas in this paper remain unchanged regardless of the point of transition. Extended tables, utilizing mid-year transitions, appear in another paper in the current issue of this journal (Skoog and Ciecka, 2001).

4. The reader will notice in (6a) that we no longer use concepts nor the notation from demography to define worklike expectancy. See Skoog (2002) for the proof that the mean of the years-of-activity random variable is equal to its counterpart expressed in demography notation.

5. Related results, including work-life expectancy, exist for those inactive at age x.

6. The means in table 1 are approximately .5 years smaller than the work-life expectancies reported by Ciecka, Donley, and Goldman. This is due to two reasons: (1) their calculations utilize mid-year transitions and (2) they use the increment-decrement model to age 75 and abandon the increment-decrement model for later ages in favor of an assumption of labor force activity that is proportional to participation rates from ages 76-80.

7. A 50% probability interval may be one way to explicate the legal idea of "accuracy to within a reasonable degree of economic certainty" and would be consistent with the interpretation that something is more likely

true than not true.

8. Here, and in subsequent examples, we ignore interpolation between years of activity; so the probability intervals are approximate. We choose to do this rather than become sidetracked with interpolation issues that would be a distraction from the main results of the paper.

9. This follows because the mean is less than the median and because these characteristics are defined over 100% probability intervals.

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