MARKOV WORK LIFE TABLE RESEARCH IN THE UNITED STATES

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1. INTRODUCTION

Prior to 1982, work life tables in the United States could be viewed as the labor force counterpart of life tables. Most work in this area emanated from the US Bureau of Labor Statistics (BLS) and was based on the assumptions that men entered and left the labor force only once in their lives and women only entered and left the labor force as a result of a change in their marital or parental status. The work life model for men especially was demographic in nature since departure from the labor force was akin to death in a life table in the sense that labor force reentry was not possible, just as reentry into a life table cannot occur after death. We now refer to this type of construct as the *conventional model* of work life. Tables produced by Fullerton and Byrne (1976), using data from 1970, illustrate this approach to work life expectancy (WLE).

The BLS broke away from the conventional model in 1982 and 1986 (US Bureau of Labor Statistics, 1982, 1986) when it published work life tables based on a *Markov process*, or *Increment–Decrement model*. In Bulletin 2135, the BLS viewed both men and women as "entering and leaving the labor market repeatedly during their lifetimes, with nearly all participating

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for some period during their lives." Tables were based on gender, age, initial labor force status (i.e., initially active in the labor force, inactive, and a blend of the two), and either educational attainment or race. Bulletin 2135, based on 1977 data, contained the BLS's first Markov work life table, but it also contained the most complete exposition of the conventional model, as well as some WLEs computed with the conventional model. The 1986 table in Bulletin 2254, based on 1979–1980 data, was the last BLS or other US government agency prepared WLE table; it contained only Markov process–generated tables. Others (primarily Ciecka, Donley, & Goldman, 2000; Skoog & Ciecka, 2001a, 2001b; Millimet, Nieswiadomy, Ryu, & Slottje, 2003; Krueger, 2004) have produced updated WLE tables with essentially the same method as pioneered by the BLS in its 1882 and 1986 bulletins.

A relatively small minority of forensic economists have used the *LPE model* of labor market activity. In this model, one multiplies L (probability of survival) by P (probability of participating in the labor force) and by E (probability of employment). The LP part of this model is, in effect, another alternative to the conventional model and the Markov model of work life. We discuss relations among the Markov, conventional, and LPE models in Section 2. Section 3 contains a discussion of the most recent Markov process–related work in the United States. Section 4 presents the theory and an example of an occupational-specific WLE table. Section 5 contains a brief comparison between US and UK work life–related research. We conclude with some ideas for future research in Section 6.

2. MARKOV, CONVENTIONAL, AND LPE MODELS

Transition probabilities comprise the primitive terms in a Markov process model. We let ${}^m p_x^n$ denote the probability that a person in state m at age x will be in state n at age $x+1$ where $m = \{a, i\}$ and $n = \{a, i, d\}$ and where a stands for active in the labor force, i for inactive, and d for the death state. We assume

$$\begin{aligned} {}^a p_x^a + {}^a p_x^i + {}^a p_x^d &= 1 \\ {}^i p_x^a + {}^i p_x^i + {}^i p_x^d &= 1 \end{aligned} \tag{1}$$

and that the transition to state d does not depend on being active or inactive, i.e., ${}^a p_x^d = {}^i p_x^d = {}^\bullet p_x^d$. Let ${}^a l_x$ and ${}^i l_x$ denote the *number* of actives and inactives at age x in a specific group (usually defined by gender, education, and initial labor force status). The magnitudes of ${}^{a}l_x$and ${}^{i}l_x$ correspond to the active and inactive portions of a population if we desire WLE regardless of initial state. If instead we desire work life for initial actives, let ${}^{a}l_x$ equal a radix value (usually taken to be 100,000) and set ${}^{i}l_x = 0$. Conversely, for work life for initial inactives, set ${}^{a}l_x = 0$ and ${}^{i}l_x = 100,000$. The Markov process model utilizes the recursions in

$$\begin{aligned} {}^{a}l_{x+1} &= {}^{a}p_x^{a\,a}l_x + {}^{i}p_x^{a\,i}l_x \\ {}^{i}l_{x+1} &= {}^{a}p_x^{i\,a}l_x + {}^{i}p_x^{i\,i}l_x \end{aligned} \tag{2}$$

If transitions occur uniformly throughout the year between age x and $x+1$, then

$$L_x^a = .5({}^{a}l_x + {}^{a}l_{x+1}) \tag{3}$$

captures person-years of activity between ages x and $x+1$. WLE without regard to initial labor force status at age x becomes

$${}^{\bullet}e_x^a = \sum_{j=x}^{\mathrm{TA}-1} \frac{L_j^a}{({}^{a}l_x + {}^{i}l_x)} \tag{4a}$$

where TA denotes the youngest age at which everyone in the population has died. Using ${}^{a}l_x = 100,000$ and ${}^{i}l_x = 0$ in Eq. (2), work life for initial actives is obtained as follows:

$${}^{a}e_x^a = \sum_{j=x}^{\mathrm{TA}-1} \frac{L_j^a}{{}^{a}l_x} \tag{4b}$$

Using ${}^{a}l_x = 0$ and ${}^{i}l_x = 100,000$ in Eq. (2), work life for initial inactives is obtained as follows:

$${}^{i}e_x^a = \sum_{j=x}^{\mathrm{TA}-1} \frac{L_j^a}{{}^{i}l_x} \tag{4c}$$

The Markov model places no restrictions of transition probabilities beyond being nonnegative and fulfilling Eq. (1). However, Skoog and Ciecka (2004b) have shown that both the conventional model and the LP part of the LPE model are in fact Markov models themselves but with additional restrictions imposed on transition probabilities. To see this, let l_x denote the number of people alive at age x, let $L_x = .5(l_x + l_{x+1})$ be the average number alive between ages x and $x+1$, and let pp_x denote the labor force participation rate at age x. Then the conventional model requires

$$
\begin{aligned}
{}^a p_x^a &= \left(\frac{L_{x+1}}{L_x}\right) \\
{}^a p_x^i &= 0 \\
{}^i p_x^a &= \left(\frac{L_{x+1}}{L_x}\right)\left(\frac{(pp_{x+1} - pp_x)}{(1 - pp_x)}\right) \\
{}^i p_x^i &= \left(\frac{L_{x+1}}{L_x}\right)\left(1 - \frac{(pp_{x+1} - pp_x)}{(1 - pp_x)}\right)
\end{aligned}
\tag{5a}
$$

before the age of peak labor force participation, and it requires

$$
\begin{aligned}
{}^a p_x^a &= \left(\frac{L_{x+1}}{L_x}\right)\left(\frac{pp_{x+1}}{pp_x}\right) \\
{}^a p_x^i &= \left(\frac{L_{x+1}}{L_x}\right)\left(1 - \frac{pp_{x+1}}{pp_x}\right) \\
{}^i p_x^a &= 0 \\
{}^i p_x^i &= \left(\frac{L_{x+1}}{L_x}\right)
\end{aligned}
\tag{5b}
$$

beyond the age of postpeak labor force participation.[1]

The LP part of the LPE model requires

$$
\begin{aligned}
{}^a p_x^a = {}^i p_x^a &= \left(\frac{L_{x+1}}{L_x}\right) pp_x \\
{}^a p_x^i = {}^i p_x^i &= \left(\frac{L_{x+1}}{L_x}\right)(1 - pp_x)
\end{aligned}
\tag{6}
$$

These extremely restrictive assumptions make little sense. For example, the conventional model for men requires that nobody leaves the labor force for any reason other than death (i.e., ${}^a p_x^i = 0$) prior to peak labor force participation that occurs at about age 34, and it completely disallows labor force entry after the age of peak labor force participation (i.e., ${}^i p_x^a = 0$). The LPE model does not recognize that labor force status at age x tells us anything whatsoever about status at age $x + 1$ (i.e., ${}^a p_x^a = {}^i p_x^a$ and ${}^a p_x^i = {}^i p_x^i$). These severe restrictions are wrong on their face and certainly are disconfirmed by estimates of transition probabilities (see Krueger, 2004 for estimates of transition probabilities that show that these assumptions are wrong).

The BLS properly abandoned the conventional model in Bulletin 2135 in favor of the more general Markov model, here summarized by Eqs. (1)–(4c). This model, without the restrictive assumptions in Eqs. (5a) through (5c), dominates the conventional and the LPE models theoretically, empirically, and in use. Skoog and Ciecka (2004b) conclude that "... there is every reason to embrace the Markov model until it too is dominated by a superior model." In what follows, the term *Markov model* refers to the unrestricted version that is free of the assumptions in Eqs. (5a), (5b), and (6) even though the conventional model and the LPE model are, strictly speaking, Markov models themselves.

3. MARKOV WORK LIFE TABLE RESEARCH IN THE UNITED STATES

Prior to 2001, the primary object of Markov model work, as with the conventional model, was to produce a single expected value, WLE, given a person's gender, age, initial labor force status, and either education or race. Skoog and Ciecka (2001a, 2001b, 2002, 2003) were able to capture the Markov model's probabilistic implications by viewing years of activity (YA) and years to final labor force separation (YFS) as random variables. This allowed them to determine entire probability mass functions (pmf's) for YA and YFS and move beyond the study of expectations. To explain this approach, let $YA_{x,m}$ denote the years-of-activity random variable with $p_{YA}(x, m, y)$ being the probability that a person who is in state m at exact age x will accumulate $YA_{x,m} = y$ years of labor force activity in the future. In a similar vein, let $YFS_{x,m}$ denote the years-to-final-separation random variable where $p_{YFS}(x, m, y)$ represents the probability that a person who is in state m at exact age x makes a final separation from the labor force in $YFS_{x,m} = y$ years. YA and YFS differ in that the former only counts time in the labor force; the latter counts all time, including inactive time, prior to final departure from the labor force.

The YA and YFS probability mass functions with midperiod transitions for initial actives and inactives are specified below by the combination of global conditions, boundary conditions, and main recursions. With pmf's in hand, measurement of labor market activity was not limited to expected values (like WLE); other measures of central tendency such as properly computed medians (often used for final labor force separation in the case of YFS) and modes could be computed. Measures of dispersion and shape like the standard deviation, skewness, and kurtosis can be computed as well. In addition, probability intervals of various sizes can be calculated. The 50% probability interval may be of particular importance since it corresponds to the idea of accuracy to within a reasonable degree of economic certainty, a critical concept when providing expert testimony. In short, we know the entire probability distribution implied by the Markov model given gender, age, initial labor force status, and education. Skoog and Ciecka provided 24 tables for YA and another 24 tables for YFS characteristics – for six education groups for two initial labor force states for each gender. Tables 1 and 2, for initially active men with a high school diploma only, illustrate the most important characteristics captured by the pmf's.[2] A separate pmf underlies every row in Tables 1 and 2 and in all of 24 tables published by Skoog and Ciecka. As an illustration, Figs. 1 and 2 show pmf's for 30-year-old initially active men with high school diploma only. In Table 1, initially active 30-year-old men have a WLE of 28.26 years, and a median and mode of 28.90 and 30.50, respectively, with a distribution that is slightly skewed to the left (skewness coefficient of –.66) and a bit leptokurtic (kurtosis coefficient of 3.60). The standard deviation of YA is 8.36 years, implying a coefficient of variation of approximately .30. The smallest interval containing 50% of the probability is 26.50 years on the low side and 35.28 years on the high side, whereas the interquartile range is from 23.17 to 33.35 years. The interval which excludes 10% of the probability in each tail of the

Table 1. Skoog/Ciecka Years of Activity Characteristics for Initially Active Men with a High School Diploma Only.

Age	WLE						Minimal 50% PI					
	Mean	Median	Mode	SD	SK	KU	Low	High	25th%	75%	10%	90%
30	28.26	28.90	30.50	8.36	−0.66	3.60	26.50	35.28	23.17	33.35	16.49	37.27
31	27.40	28.01	29.50	8.25	−0.63	3.52	25.50	34.20	22.33	32.43	15.73	36.33
32	26.55	27.11	28.50	8.14	−0.60	3.45	24.50	33.13	21.50	31.50	15.00	35.39
33	25.71	26.22	27.50	8.02	−0.56	3.39	23.50	32.05	20.67	30.58	14.28	34.45
34	24.86	25.33	27.50	7.90	−0.53	3.32	23.03	31.50	19.84	29.66	13.57	33.51
35	24.02	24.45	26.50	7.78	−0.50	3.26	22.11	30.50	19.01	28.74	12.85	32.59
36	23.18	23.56	25.50	7.66	−0.46	3.20	21.21	29.50	18.20	27.82	12.15	31.66
37	22.35	22.67	24.50	7.53	−0.42	3.15	20.31	28.50	17.40	26.90	11.49	30.73
38	21.53	21.78	23.50	7.40	−0.39	3.10	19.41	27.50	16.61	25.99	10.82	29.80
39	20.71	20.90	22.50	7.26	−0.35	3.06	18.50	26.47	15.82	25.07	10.18	28.88
40	19.89	20.02	21.50	7.12	−0.31	3.02	17.64	25.50	15.04	24.16	9.57	27.95

Source: Skoog and Ciecka (2001b).

Table 2. Skoog/Ciecka Years to Final Separation Characteristics for Initially Active Men with a High School Diploma Only.

Age	YFSE						Minimal 50% PI					
	Mean	Median	Mode	SD	SK	KU	Low	High	25th%	75%	10%	90%
30	33.69	33.47	33.50	10.37	−0.42	3.70	28.50	38.94	28.09	39.54	20.00	45.95
31	32.76	32.49	32.50	10.27	−0.38	3.63	27.50	37.91	27.14	38.55	19.18	44.96
32	31.83	31.52	31.50	10.16	−0.34	3.57	26.50	36.87	26.19	37.57	18.37	43.97
33	30.91	30.54	30.50	10.05	−0.30	3.50	25.50	35.83	25.24	36.59	17.56	42.99
34	29.98	29.56	29.50	9.95	−0.26	3.44	24.50	34.80	24.30	35.61	16.76	42.00
35	29.06	28.59	28.50	9.84	−0.22	3.39	23.50	33.76	23.36	34.64	15.96	41.01
36	28.14	27.62	27.50	9.73	−0.18	3.33	22.50	32.71	22.42	33.66	15.17	40.03
37	27.23	26.64	26.50	9.62	−0.14	3.28	21.50	31.67	21.49	32.68	14.39	39.04
38	26.31	25.67	25.50	9.51	−0.09	3.24	20.50	30.62	20.55	31.71	13.61	38.06
39	25.40	24.71	24.50	9.40	−0.05	3.19	19.50	29.57	19.61	30.74	12.84	37.08
40	24.50	23.74	23.50	9.29	−0.01	3.15	18.50	28.52	18.67	29.76	12.08	36.10

Source: Skoog and Ciecka (2003), with permission from National Association Forensic Economics.

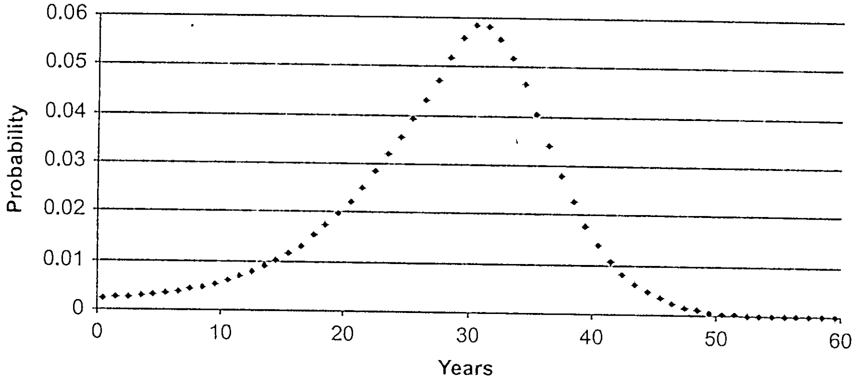

Fig. 1. PMF for Years of Activity for Initially Active 30-Year-Old Men with High School Diploma Only.

distribution ranges from 16.49 to 37.27 years. From Table 2, we know that initially active 30-year-old men with only a high school education have an average of 33.69 years until final separation from the labor force, approximately 5.4 years more than WLE. The remainder of the age-30 row gives the other characteristics of years to final separation as Table 1 does for years of labor force activity.

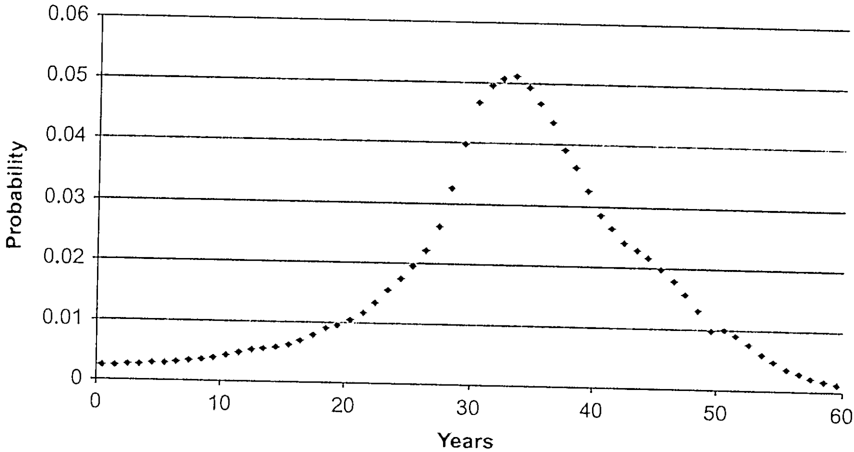

Fig. 2. PMF for Years to Final Labor Force Separation for Initially Active 30-Year-Old Men with High School Diploma Only.

Global Conditions for Random Variables $\mathrm{RV} \in \{\mathrm{YA}, \mathrm{YFS}\}$ with Midpoint Transitions

$$p_{\mathrm{RV}}(x,a,y) = p_{\mathrm{RV}}(x,i,y) = 0 \quad \text{if } y<0 \text{ or } y>\mathrm{TA}-x-.5$$

$$p_{\mathrm{RV}}(\mathrm{TA},a,0) = p_{\mathrm{RV}}(\mathrm{TA},i,0) = 1$$

$${}^{a}p_{x}^{d} = {}^{i}p_{x}^{d} = 1 \quad \text{for} \quad x \geq \mathrm{TA}-1$$

YA Probability Mass Functions for $\mathrm{YA}_{x,m} = y$ for $m \in \{a,i\}$ with Midpoint Transitions

Boundary Conditions

$$p_{\mathrm{YA}}(x,a,0) = 0$$

$$p_{\mathrm{YA}}(x,a,.5) = {}^{a}p_{x}^{d} + {}^{a}p_{x}^{i} p_{\mathrm{YA}}(x+1,i,0)$$

$$p_{\text{YA}}(x,i,0) = {}^{i}p_x^d + {}^{i}p_x^i p_{\text{YA}}(x+1,i,0)$$

for $x = \text{BA}, \ldots, \text{TA} - 1$

Main Recursions

$$p_{\text{YA}}(x,a,y) = {}^{a}p_x^a p_{\text{YA}}(x+1,a,y-1) + {}^{a}p_x^i p_{\text{YA}}(x+1,i,y-.5),$$
$$y = 1.5, 2.5, 3.5, \ldots, \text{TA} - x - .5$$

$$p_{\text{YA}}(x,i,y) = {}^{i}p_x^a p_{\text{YA}}(x+1,a,y-.5) + {}^{i}p_x^i p_{\text{YA}}(x+1,i,y),$$
$$y = 1, 2, 3, \ldots, \text{TA} - x - .5$$

for $x = \text{BA}, \ldots, \text{TA} - 1$

YFS Probability Mass Functions for $\text{YFS}_{x,m} = y$ for $m \in \{a,i\}$ with Midpoint Transitions

Boundary Conditions

$$p_{\text{YFS}}(x,a,y) = 0, \quad y = 0, 1, 2, 3, \ldots, \text{TA} - 1$$

$$p_{\text{YFS}}(x,i,y) = 0, \quad y = .5, 1, 2, 3, \ldots, \text{TA} - 1$$

$$p_{\text{YFS}}(x,a,.5) = {}^{a}p_x^d + {}^{a}p_x^i p_{\text{YFS}}(x+1,i,0)$$

$$p_{\text{YFS}}(x,i,0) = {}^{i}p_x^d + {}^{i}p_x^i p_{\text{YFS}}(x+1,i,0)$$

for $x = \text{BA}, \ldots, \text{TA} - 1$

Main Recursions

$$p_{\text{YFS}}(x,a,y) = {}^{a}p_x^a p_{\text{YFS}}(x+1,a,y-1) + {}^{a}p_x^i p_{\text{YFS}}(x+1,i,y-1)$$

$$p_{\text{YFS}}(x,i,y) = {}^{i}p_x^a p_{\text{YFS}}(x+1,a,y-1) + {}^{i}p_x^i p_{\text{YFS}}(x+1,i,y-1)$$

for $x = \text{BA}, \ldots, \text{TA} - 1$ and $y = 1.5, 2.5, 3.5, \ldots, \text{TA} - x - .5$

Skoog and Ciecka (2004b) also bootstrapped estimates of standard errors for WLE and other characteristics illustrated in Table 1. Bootstrapped standard errors refer to WLE expectancy itself and other YA characteristics due to sampling error, whereas the standard deviations illustrated in Table 1 refer to intrinsic variation in YA in the population itself. Bootstrap standard errors are much smaller than the standard deviations in Table 1. The former give us information about the accuracy of estimates of WLE (and other characteristics), whereas the latter refer to the range of years of labor market activity. Table 3, for all initially active men, illustrates bootstrapped standard errors. The bootstrap standard error for WLE is .20 years for initially active 30-year-old men in Table 3. That is, the sampling error is much smaller than the standard deviation of YA itself, which was found to be 8.36 years in Table 1.

Millimet, Nieswiadomy, Ryu, and Slottje (MNRS, 2003) use data from 1992 to 2000 to estimate WLEs using the BLS approach to a Markov process outlined above, and they also estimate parametric models of transition probabilities using logit functions. Referring to the BLS and logit approaches, they indicate that "[f]or both males and females the work life expectancies are extremely close ... at all education and age levels." MNRS also estimate a multinomial logit function that they use to get work life estimates for those initially employed, unemployed, and inactive. They have two main conclusions: (1) for both men and women with less than a high school education, work life estimates for the unemployed are closer to work life estimates for inactives than for those employed and (2) as education increases, work life estimates for the unemployed approach work life estimates for employed people. Table 4 illustrates their work life tables for employed, unemployed, and inactive men.[3] The standard errors for WLE initially active men with only high school education are about the same size as the bootstrap standard errors reported by Skoog and Ciecka (see Table 3) but much smaller than the standard deviations of years of population activity in Table 1.

Krueger (2004) published WLEs by age, gender, labor force status, and education using 1998–2004 data. Table 5 illustrates some of Krueger's results.[4] Krueger's WLEs are quite close to those reported by Skoog and Ciecka (compare WLEs in Table 1 with the active column in Table 5). Krueger also provided counts of people in initial active and inactive states and the numbers that moved from one state to another by age, gender, and education. This information enables anyone to compute transition probabilities that are basic to a Markov process. Besides being current, Krueger, as other researchers, utilized the *Current Population Survey*, the

Table 3. Skoog/Ciecka Bootstrap Estimates of the Mean and Standard Deviation of Years of Activity Characteristics for Initially Active Men, Regardless of Education.

Age	Bootstrap Mean of WLE	Bootstrap SD of WLE	Bootstrap Mean of Median	Bootstrap SD of Median	Bootstrap Mean of Mode	Bootstrap SD of Mode
30	29.35	0.20	29.97	0.20	31.77	0.84
31	28.48	0.20	29.05	0.20	30.82	0.85
32	27.61	0.19	28.14	0.20	29.88	0.86
33	26.75	0.19	27.23	0.20	28.92	0.87
34	25.89	0.19	26.32	0.20	27.97	0.87
35	25.03	0.19	25.41	0.20	27.03	0.88
36	24.17	0.19	24.50	0.20	26.07	0.88
37	23.32	0.19	23.59	0.19	25.11	0.88
38	22.47	0.19	22.68	0.19	24.15	0.89
39	21.62	0.19	21.78	0.19	23.18	0.89
40	20.77	0.19	20.87	0.19	22.22	0.90

Age	Bootstrap Mean of SD	Bootstrap SD of SD	Bootstrap Mean of Skewness	Bootstrap SD of Skewness	Bootstrap Mean of Kurtosis	Bootstrap SD of Kurtosis
30	8.19	0.10	−0.81	0.04	4.02	0.10
31	8.08	0.10	−0.78	0.04	3.92	0.10
32	7.96	0.10	−0.74	0.04	3.83	0.10
33	7.84	0.10	−0.70	0.04	3.74	0.09
34	7.72	0.10	−0.66	0.04	3.65	0.09
35	7.59	0.10	−0.62	0.04	3.57	0.08
36	7.46	0.10	−0.58	0.04	3.49	0.08
37	7.33	0.10	−0.54	0.04	3.41	0.08
38	7.20	0.10	−0.49	0.04	3.34	0.07
39	7.07	0.10	−0.45	0.04	3.27	0.07
40	6.94	0.10	−0.40	0.04	3.20	0.07

Age	Bootstrap Mean of 25th%	Bootstrap SD of 25th Percentile	Bootstrap Mean of 75th%	Bootstrap SD of 75th Percentile	Bootstrap Mean of 10th%	Bootstrap SD of 10th Percentile	Bootstrap Mean of 90th%	Bootstrap SD of 90th Percentile
30	24.69	0.25	34.27	0.21	18.11	0.32	38.04	0.25
31	23.82	0.25	33.33	0.21	17.35	0.32	37.09	0.24
32	22.95	0.25	32.40	0.21	16.60	0.31	36.15	0.24
33	22.10	0.25	31.46	0.21	15.85	0.31	35.20	0.24
34	21.25	0.25	30.52	0.21	15.13	0.30	34.25	0.24
35	20.40	0.24	29.59	0.21	14.41	0.30	33.31	0.24
36	19.55	0.24	28.65	0.21	13.69	0.29	32.36	0.24
37	18.71	0.24	27.72	0.21	12.99	0.29	31.41	0.25
38	17.87	0.23	26.79	0.21	12.29	0.29	30.46	0.25
39	17.03	0.24	25.85	0.21	11.60	0.28	29.52	0.25
40	16.21	0.24	24.92	0.21	10.93	0.27	28.57	0.25

Source: Skoog and Ciecka (2004a), with permission from National Association of Forensic Economics.

Table 4. Millimet/Nieswiadomy/Ryu/Slottje Work Life Expectancies and Standard Errors for Initially Active Men with High School Diploma Only.

Age	WLE	Standard Error
30	29.477	.267
31	28.606	.266
32	27.735	.264
33	26.862	.263
34	25.989	.262
35	25.112	.261
36	24.240	.259
37	23.370	.258
38	22.504	.256
39	21.641	.254
40	20.782	.252

Source: Millimet, Nieswiadomy, Ryu, and Slottje (2003).

Table 5. Krueger's Work Life Expectancies for Initially Active, Inactive, and All Men with a High School Diploma Only.

Age	All	Inactive	Active
30	28.46	26.35	28.64
31	27.63	25.46	27.77
32	26.74	24.55	26.91
33	25.88	23.72	26.05
34	25.01	22.83	25.20
35	24.10	21.72	24.34
36	23.24	20.75	23.48
37	22.43	19.87	22.63
38	21.52	18.88	21.77
39	20.69	17.92	20.93
40	19.80	16.88	20.09

Source: Krueger (2004), with permission from National Association of Forensic Economics.

same data used by the BLS to compile labor statistics. Among other attributes, his dataset replicates US participation rates reported by the BLS, which gives users a high degree of confidence in his transition probabilities. Forensic economists can use Krueger's WLEs directly from his tables, but the transition probabilities themselves allow users to compute all of the individual terms in Eqs. (4a)–(4c). These terms, when summed, comprise

WLE. However, each individual term may be of interest when calculating the present value of lost earnings because successive individual terms are subject to greater discounting. For example, using Eq. (4b), individual terms are $L_x^a/{}^al_x, L_{x+1}^a/{}^al_x, ..., L_{\mathrm{TA}-1}^a/{}^al_x$ with present values of $(L_x^a/{}^al_x)(1+r)^{-.5}, (L_{x+1}^a/{}^al_x)(1+r)^{-1.5}, ..., (L_{\mathrm{TA}-1}^a/{}^al_x)(1+r)^{-(\mathrm{TA}-x-.5)}$ for lost earnings of one dollar at ages x, $x+1,...,$ TA–1 using r for the net discount rate and assuming midperiod receipt of earnings.[5] Krueger also announced his intention to make transition probabilities available on a rolling 10-year basis. His most currently available data covers the period 1998–2007.

Krueger, Skoog, and Ciecka (2006) calculated WLEs for full-time and part-time workers based on the Markov model. This research provides the first estimates of the percentage of work life spent in full-time activity and part-time activity. It also shows differences in work life for those initially in full-time activity and part-time work. The underlying Markov theory resembles Eqs. (1)–(4c). Using *ft*, *pt*, and *i* for full time, part time, and inactive, respectively, we have

$$\begin{aligned} {}^{ft}p_x^{ft} + {}^{ft}p_x^{pt} + {}^{ft}p_x^{i} + {}^{ft}p_x^{d} &= 1 \\ {}^{pt}p_x^{ft} + {}^{pt}p_x^{pt} + {}^{pt}p_x^{i} + {}^{pt}p_x^{d} &= 1 \\ {}^{i}p_x^{ft} + {}^{i}p_x^{pt} + {}^{i}p_x^{i} + {}^{i}p_x^{d} &= 1 \end{aligned} \tag{7}$$

The left superscript indicates beginning period status at age x, and the right superscript indicates the status at the end of the period. We assume that ${}^{ft}p_x^d = {}^{pt}p_x^d = {}^{i}p_x^d = {}^{\bullet}p_x^d$, i.e., the probability of dying is independent of labor force status. If one wishes WLE conditional upon an initial status, say full-time, set ${}^{ft}l_x$ to 100,000, and ${}^{pt}l_x = 0$ and ${}^{i}l_x = 0$ to start the recursions in Eq. (8) and calculate the number of persons in the statuses on the left-hand sides at age $x+1$:

$$\begin{aligned} {}^{ft}l_{x+1} &= {}^{ft}p_x^{ft}\,{}^{ft}l_x + {}^{pt}p_x^{ft}\,{}^{pt}l_x + {}^{i}p_x^{ft}\,{}^{i}l_x \\ {}^{pt}l_{x+1} &= {}^{ft}p_x^{pt}\,{}^{ft}l_x + {}^{pt}p_x^{pt}\,{}^{pt}l_x + {}^{i}p_x^{pt}\,{}^{i}l_x \\ {}^{i}l_{x+1} &= {}^{ft}p_x^{i}\,{}^{ft}l_x + {}^{pt}p_x^{i}\,{}^{pt}l_x + {}^{i}p_x^{i}\,{}^{i}l_x \end{aligned} \tag{8}$$

Define

$${}^{ft}L_x = .5({}^{ft}l_x + {}^{ft}l_{x+1}) \tag{9}$$

as the person-years spent in the full-time state, and calculate

$$ {}^{ft}e_x^{ft} = \sum_{j=x}^{j=\mathrm{TA}-1} \frac{{}^{ft}L_j}{{}^{ft}l_x} \tag{10} $$

as the WLE of years in the full-time state (the upper right ft superscript) having started in the ft state (the upper left superscript) for a person exact age x.

To count time in the part-time state, but again starting in the full-time state, we compute

$$ {}^{pt}L_x = .5({}^{pt}l_x + {}^{pt}l_{x+1}) \tag{11} $$

as the person-years spent in the part-time activity. We calculate the WLE of part-time years, starting full time, as

$$ {}^{ft}e_x^{pt} = \sum_{j=x}^{j=\mathrm{TA}-1} \frac{{}^{pt}L_j}{{}^{ft}l_x} \tag{12} $$

In this way, overall WLE from the full-time state is defined by the sum of the time in the active states, full-time and part-time, as

$$ {}^{ft}e_x^{a} \equiv {}^{ft}e_x^{ft} + {}^{ft}e_x^{pt} \tag{13} $$

Had we begun in the part-time state, we would have started Eq. (8) with ${}^{ft}l_x = 0$, ${}^{pt}l_x = 100,000$, and ${}^{i}l_x = 0$ and calculated ${}^{pt}e_x^{ft}$ and ${}^{pt}e_x^{pt}$. If inactivity had been the initial state, we would have assumed ${}^{ft}l_x = 0$, ${}^{pt}l_x = 0$, and ${}^{i}l_x = 100,000$ and calculated ${}^{i}e_x^{ft}$ and ${}^{i}e_x^{pt}$.

Table 6 illustrates some Krueger/Skoog/Ciecka results for part-time/full-time activity for males with high school diploma only. For example, consider 30-year-old males who are employed full time. The expected number of years in future part-time work is 2.12 years, and the expected time in full-time activity is 26.53 years. Total expected labor force time would be 28.65 years ($=2.12+26.53$), and approximately .93 ($=26.53/28.65$) of labor force activity would be in the full-time state (see the last column of Table 6).

In a recent paper, Skoog and Ciecka (2008) presented estimates of the mean and other distributional characteristics of the present value random variable evaluated at several net discount rates. This research contains the first tabulations of the mean, median, standard deviation, skewness, kurtosis, and probability intervals for present value functions at various

Table 6. Krueger/Skoog/Ciecka Full-time and Part-time Expectancies for Males with High School Diploma Only.

Age	Years of NILF				Years Part-time				Years Full-time				Years in the Labor Force				Portion Years Full-time			
	Beginning State				Beginning State				Beginning State				Beginning State				Beginning State			
	All	NILF	PT	FT	All	NILF	PT	FT	All	NILF	PT	FT	All	NILF	PT	FT	All	NILF	PT	FT
30	17.86	20.03	17.97	17.66	2.16	2.22	3.05	2.12	26.30	24.07	25.30	26.53	28.46	26.29	28.35	28.66	0.92	0.92	0.89	0.93
31	17.74	19.97	18.16	17.58	2.13	2.18	2.87	2.09	25.51	23.23	24.35	25.71	27.64	25.41	27.22	27.80	0.92	0.91	0.89	0.92
32	17.71	19.93	18.13	17.51	2.10	2.14	2.95	2.06	24.63	22.38	23.36	24.88	26.74	24.52	26.31	26.94	0.92	0.91	0.89	0.92
33	17.63	19.82	17.88	17.44	2.09	2.09	3.03	2.04	23.78	21.60	22.60	24.03	25.87	23.69	25.63	26.07	0.92	0.91	0.88	0.92
34	17.57	19.78	17.76	17.36	2.05	2.07	3.01	2.02	22.96	20.73	21.81	23.19	25.01	22.80	24.82	25.21	0.92	0.91	0.88	0.92
35	17.55	19.97	17.83	17.29	2.02	2.06	2.90	1.99	22.08	19.61	20.92	22.36	24.10	21.67	23.82	24.35	0.92	0.90	0.88	0.92
36	17.48	20.03	17.70	17.22	2.00	2.05	2.80	1.96	21.24	18.64	20.23	21.53	23.24	20.69	23.02	23.50	0.91	0.90	0.88	0.92
37	17.37	19.99	17.62	17.15	1.97	2.02	2.82	1.93	20.46	17.79	19.35	20.71	22.43	19.81	22.17	22.64	0.91	0.90	0.87	0.91
38	17.36	20.05	17.69	17.08	1.94	2.00	2.83	1.91	19.57	16.82	18.35	19.88	21.52	18.82	21.18	21.79	0.91	0.89	0.87	0.91
39	17.27	20.11	17.66	17.01	1.92	2.01	2.81	1.88	18.77	15.84	17.49	19.07	20.69	17.85	20.30	20.95	0.91	0.89	0.86	0.91
40	17.25	20.24	17.54	16.94	1.90	1.99	2.82	1.86	17.89	14.82	16.69	18.25	19.80	16.81	19.51	20.11	0.90	0.88	0.86	0.91

Note: NILF, not in labor force; FT, full time; PT, part time.
Source: Krueger, Skoog, and Ciecka (2006), with permission from National Association of Forensic Economics.

net discount rates. Table 7 shows these characteristics for 30-year-old men, regardless of education, at net discount rates of 0, .005, .01, .0125, .0150, .0175, .02, .025, .03, .035, and .04. The row with the net discount rate of zero gives WLE and the distributional characteristics of years of labor force activity for men age 30. The table shows negative skewness at all net discount rates, with the median exceeding the mean present value. Standard deviations vary inversely with discount rates, and probability intervals tighten (e.g., see the interquartile range) as net discount rates increase. The Ogden tables, discussed in Section 5, contain expected present values at the legally mandated rate of 2.5% in Great Britain. However, none of the distributional characteristics in Table 7 have been estimated for the present value random variable in any previous work in the United States or elsewhere.

4. OCCUPATION-SPECIFIC WORK LIVES

In most applications, forensic economists assume, implicitly or explicitly, that all future work life will be in the occupation used to determine the earnings base. While this is reasonable in many applications, we know that this would not be a good assumption for workers in some occupations. For example, professional baseball and football players do not continue in these occupations as would be suggested by the WLE table for the overall relevant population. While many such persons may continue in their sport in a related role, such as a coach, scout, or announcer, the time spent in competitive play is drastically shorter than the years dictated by one of the population work life tables, and competent forensic economists would adjust their estimates accordingly. However, there are less obvious situations, such as railroad workers (shorter work lives than average) and forensic economists (likely longer work lives than average). Because reliable occupation- or industry-specific data and technically trained economists, statisticians, demographers, or actuaries are needed to produce such tables on a case-by-case basis, there are relatively few such tables currently available.

Skoog and Ciecka (2006a) used multiple decrement theory, known as competing risks in biometrics, to estimate WLEs and other distributional characteristics of railroad workers. The Bureau of the Actuary of the US Railroad Retirement Board collects data for all US railroad workers. In this theory, transitions into railroad work are disallowed, but death, disability, retirement, or withdrawal (to another occupation or out of the labor

Table 7. Skoog/Ciecka Characteristics of Present Value Probability Mass Functions for Initially Active Men, Regardless of Education.

Age	Net Discount Rate	Expected Present Value	Median Present Value	Standard Deviation of Present Value	Skewness of Present Value	Kurtosis of Present Value	10th Percentile Present Value	25th Percentile Present Value	75th Percentile Present Value	90th Percentile Present Value
30	0.0000	30.00	31.50	8.08	−0.69	3.91	19.50	25.50	35.50	39.50
30	0.0050	27.54	28.44	7.07	−0.81	4.14	18.42	23.90	32.38	35.41
30	0.0100	25.37	26.31	6.22	−0.93	4.40	17.25	22.28	29.58	31.98
30	0.0125	24.38	25.39	5.85	−0.99	4.55	16.67	21.49	28.28	30.60
30	0.0150	23.45	24.48	5.50	−1.05	4.70	16.17	20.73	27.06	29.30
30	0.0175	22.58	23.57	5.19	−1.11	4.86	15.82	19.98	25.96	28.08
30	0.0200	21.75	22.69	4.89	−1.17	5.02	15.45	19.41	24.99	26.94
30	0.0250	20.23	21.08	4.38	−1.27	5.36	14.58	18.23	23.22	24.81
30	0.0300	18.88	19.68	3.93	−1.38	5.72	13.73	17.09	21.59	22.90
30	0.0350	17.66	18.50	3.55	−1.48	6.09	13.14	16.11	20.10	21.21
30	0.0400	16.57	17.35	3.22	−1.57	6.47	12.44	15.24	18.75	19.69

Note: Krueger's (2004) transition probabilities were used for all males until age 80 and active-to-active and inactive-to-active transition probabilities were gradually diminished to zero as age approached 111.
Source: Skoog and Ciecka (2008), with permission from National Association of Forensic Economics.

force) cause members to leave the occupation. They use the following notation:

x denotes exact age;
ω denotes the youngest age for which the probability of being active in the railroad industry is zero;
s denotes years of railroad service, here $s = 0, \ldots, x - 17$;
$q'^{(1)}_x$ denotes the mortality rate between age x and $x + 1$ (this is the net rate of mortality),
$q^{(2)}_{x,s}$ denotes the probability of a railroad disability retirement between x and $x + 1$ given s years of service;
$q^{(3)}_{x,s}$ denotes the probability of a railroad age retirement between x and $x + 1$ given s years of service;
$q^{(4)}_{x,s}$ denotes the probability of withdrawal from railroad work between x and $x + 1$ given s years of service;
$q^{(1)}_x = q'^{(1)}_x[1 - .5(q^{(2)}_{x,s} + q^{(3)}_{x,s} + q^{(4)}_{x,s})]$, $q^{(1)}_x$ denotes mortality probability;
$WLE^{CR}_{x,s}$ denotes competing risks railroad WLE for an individual at age x with s years of railroad service under the assumption of mortality, disability, age retirement, and withdrawal as the competing risks.

The probability that a railroad worker age x with s years of service in railroad work will remain in the railroad industry at age $x + 1$ is

$$ {}_1p_{x,s} = 1 - (q^{(1)}_x + q^{(2)}_{x,s} + q^{(3)}_{x,s} + q^{(4)}_{x,s}) \tag{14} $$

The probability of continuing as a railroad worker is defined recursively by

$$ \begin{gathered} {}_{i+1}p_{x,s} = {}_ip_{x,s}[1 - (q^{(1)}_{x+i} + q^{(2)}_{x+i,s+i} + q^{(3)}_{x+i,s+i} + q^{(4)}_{x+i,s+i})] \\ \text{where} \quad i = 1, \ldots, \omega - x - 1 \quad \text{and} \quad {}_{\omega-x}p_{x,s} = 0 \end{gathered} \tag{15} $$

The pmf for future years of railroad work YA for a person age x with s service years in the competing risks (CR) model consists of the boundary condition and a main recursion in

$$ \begin{aligned} &\text{Boundary Condition: } p^{\text{CR}}_{\text{YA}}(x, s, .5) = 1 - {}_1p_{x,s} \\ &\text{Main Recursion: } p^{\text{CR}}_{\text{YA}}(x, s, y) = {}_{y-.5}p_{x,s} - {}_{y+.5}p_{x,s} \\ &\qquad y = 1.5, 2.5, \ldots, \omega - x - .5 \end{aligned} \tag{16} $$

Table 8 illustrates railroad WLEs and standard deviations (see the column entitled New CR Expectancy and the column immediately to its right) computed with Eqs. (14)–(16) for railroad workers ages 30, 35, and 40 with various years of service. The column in Table 8, entitled Old AAR-Type Expectancy, contains WLEs when withdrawals $q_{x,s}^{(4)}$ are excluded from Eqs. (14)–(16). This column resembles work previously published by the Association of American Railroads. One immediately notices the striking difference in work lives between the new and old estimates reflecting the importance of withdrawals from railroad work. Finally, the last two columns of Table 8 show Increment–Decrement model WLEs and standard deviations that reflect the special 30 year/age 60 retirement provisions enjoyed by railroad workers. This is a hybrid model that allows for entry and egress until age 60 but disallows reentry into railroad work after age 60.[6]

Skoog and Ciecka (2009) presented a paper that calculated work lives of position players in major league baseball. The model was the first to move beyond a first-order discrete state Markov model to a second-order model, allowing next year's state to depend on the states not only in this period but also in the previous period. The data problem is solved because information on the playing careers of baseball players is publicly available in the United States.

Table 8. Work Life Expectancies of Railroad Workers Utilizing Competing Risk (CR) Theory and the Increment–Decrement Model.

Age	Service Years	New CR Expectancy	Standard Deviation	Old AAR-Type Expectancy	ID Expectancy	Standard Deviation
30	0	14.21	12.03	27.05	28.81	6.58
30	5	18.51	11.34	26.62	26.71	5.52
30	10	20.20	10.74	26.20	26.71	5.52
35	0	13.68	11.46	24.92	24.49	6.53
35	5	16.55	9.26	22.31	24.04	5.92
35	10	17.72	8.67	21.87	22.24	5.04
35	15	18.43	8.22	21.45	22.24	5.04
40	0	12.18	9.81	20.84	20.35	6.10
40	5	15.52	8.82	20.31	20.06	6.10
40	10	14.83	6.89	17.66	19.61	5.45
40	15	15.13	6.60	17.21	17.80	4.53
40	20	15.21	6.57	16.84	17.80	4.53

Source: Skoog and Ciecka (2006a), with permission from National Association of Forensic Economics.

5. COMPARISON OF MARKOV WORK LIFE TABLE RESEARCH IN THE UNITED STATES AND UNITED KINGDOM

We take the paper by Butt, Haberman, Verrall, and Wass (BHVW, 2008), based on 1998–2003 data, to be the standard for Markov work life research in the United Kingdom. That paper is an important contribution to the literature and rightfully focuses on a Markov process model embedded in Ogden-type tables. When comparing their work with US research, the main differences occur in regard to the information embedded into WLE. In the United Kingdom, several variables are incorporated into WLE that usually are kept separate in the United States.

In the United States, WLE typically refers to time in the labor force, whether working or looking for work. BHVW use the term more restrictively to denote time spent working – time spent unemployed being excluded from WLE. Both constructs have merit. On the one hand, we recognize that earnings and fringe benefits flow from actual work time rather than time in unemployment, and losses due to personal injury and wrongful death should capture foregone earnings and benefits. Therefore, BHVW's usage seems appropriate in personal injury and wrongful death matters. On the other hand, the US concept of WLE has merit as well. Consider a person with a work and earnings history encompassing several years. The effects of unemployment on earnings will then be incorporated into that individual's earnings base (BHVW's multiplicand), and no additional adjustment for unemployment would be required or desirable for that person. When including only employed time in WLE, one must be careful to not double count the effects of unemployment: once in a plaintiff's base earnings and a second time embedded in WLE. To avoid double counting the effects of unemployment, the forensic economist may have to "gross up" the earnings base to a full-employment equivalent before applying a BHVW-type WLE (their multiplier) that already has been diminished by the probability of unemployment.

In BHVW's construct, WLE is a present value. It would be similar to computing the present value of each term in Eqs. (4a), (4b), or (4c) and then summing all terms. This is not commonly done in the United States where WLE consists of undiscounted labor force time. The US convention potentially creates a "front loading" problem and overestimates of lost earnings (Skoog & Ciecka, 2006b). This problem is avoided in BHVW's construct of work life that focuses on present value, the ultimate object of interest for compensation purposes. However, a specific net discount rate

(2.5%) is embedded in WLE as calculated by BHVW. If, for whatever reason, a forensic economist wanted to use another discount rate, the BHVW-type WLE would have to be recomputed.

BHVW report standard errors for their WLEs. This is important because it enables users to better judge accuracy and formulate ranges within which loss estimates should lie. Similar standard errors have been computed using the US notion of WLE. In addition, standard errors have also been computed for the median, mode, standard deviation, skewness, kurtosis, and various percentile points for the US version of WLE (Skoog & Ciecka, 2004b).

BHVW incorporate disability into their WLEs. In the United States, a vocational expert often presents courtroom testimony integrated with work of a forensic economist in personal injury matters. Vocational experts are replaced in the BHVM tables by average employment experience of disabled people in the UK database. However useful this might be if only a "rough and ready" multiplier is deemed sufficient, the use of such averages generally is avoided in the United States where vocational experts assess the plaintiff for type of work and ability to hold competitive employment, postaccident. They take into account the unique qualities of the injured party – the effects of his education, training, occupation, and transferable skills. Rather than determine that a large statistical group might retain some fraction of its former capacity when disabled, a more individual analysis is undertaken. In considering transferable skills and acknowledging that the vast heterogeneity in the disabled population gives little guidance for "disabled" individuals, it is concluded that, if the injured plaintiff can hold a job postaccident and absent specific medical evidence to the contrary, economic losses are likely to be reflected in lower earnings (multiplicand) in the postaccident job rather than in lowered WLE. For example, the employment experience of "disabled" coal miners from a particular musculoskeletal injury has virtually nothing to say about what the prospective employment experience would be for an injured plaintiff schoolteacher suffering an adult onset brachial plexus injury, who further has a duty to mitigate damages by working if possible. The illusion of precision in using disability data often adds noise rather than signal, and can in fact create damages where none exists. For example, by declaring a person disabled who is earning the same amount in the same job postaccident, a statistically irrelevant lowered work life spuriously assigns damages where none may exist.

Finally, UK work life research deals only with WLE (the mean of additional years of labor force activity) whereas work in the United States has dealt with probability distributions of YA and its characteristics, including WLE.

6. POSSIBLE FUTURE RESEARCH

Future WLE-related research will likely progress along several fronts.

(1) Tables will be updated. New tables probably will be based on Krueger's 1998–2008 transition probabilities.
(2) Updated tables will likely contain more refined educational groups. For example, those with a formal high school education may be separated from GED holders. New educational groups may include only those with PhDs and other more advanced educational attainments.
(3) Expected present values of active time, as well as other distributional characteristics of the present value random variable, may be computed for a range of net discount rates.
(4) More parametric functions may be estimated and used to calculate WLE and other labor force variables.
(5) New tables and/or parametric functions may include variables like race and the impact of both race and education.
(6) We may improve on the Markov model itself, which is a discrete state autoregressive process of order one, since transition probabilities depend on a person's current labor force state but not on the path of labor force states that led to that state. An autoregressive model of order two would incorporate information on a person's current state and previous state. So, for example, a person being active at age x–1 and active at age x may lead to a different transition probability of being in a certain state at age $x+1$ than for the case of a person who was inactive at age x–1 and active at age x.

NOTES

1. Monotonicity conditions also must be fulfilled in order to get sensible estimates of transition probabilities. In Eq. (5a), ${}^{i}p_x^a$ would be negative if $pp_{x+1} < pp_x$ for ages x and $x+1$ prior to peak labor force participation. Similarly, estimated ${}^{a}p_x^i$ would be negative in Eq. (5b) if $pp_{x+1} > pp_x$ for postpeak participation rates. Since probabilities cannot be negative, the model could not be used if estimated participation rates implied negative transition probabilities.

2. Only ages 30–40 are shown. Complete sets of tables begin at ages 16, 18, 22, and 26, depending on educational attainment, through age 75.

3. Depending on educational attainment, MNRS tables start at ages 16, 17, or 21 and end at age 85.

4. Krueger supplies 10 transit tables by gender and education that can be used to compute transition probabilities. He also gives WLEs by gender and education for initial actives, inactives, and without regard to initial status. Tables begin at ages 17, 18, or 20 depending on education and run through age 75.

5. See Skoog and Ciecka (2006b) for an alternative approach that computes the present value of lost earnings using WLE and then corrects for front loading by using a set of nomograms for net discount rates of .01, .02, .03, and .04.

6. See Skoog and Ciecka (1998, 2006c) for additional estimates of Old AAR-Type expectancies and Markov process expectancies for railroad workers.

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